DAPTO HIGH SCHOOL
2007

YEAR 11
HALF-YEARLY EXAMINATION

Mathematics
Extension 1

General Instructions

* Reading Time – 5 minutes
* Working Time – 1 ½ hours
* Total Marks – 100 marks
* All questions are of equal value
* Write using a blue or black pen
* Approved calculators may be used
* All necessary working should be shown for every question
* Begin each question on a fresh sheet of paper
* Write on only one side of the paper

Question 1 — Basic Arithmetic & Algebra

1. Find the exact value of $x - \frac{1}{x}$ when $x = 2\sqrt{3} + 1$

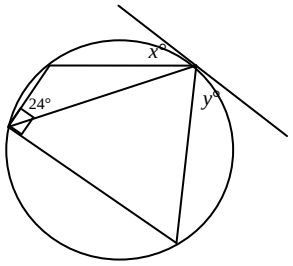
2. Simplify: $\frac{2}{x-3} - \frac{4}{x+2} - \frac{1}{x^2-x-6}$

3. Solve: $\frac{x+3}{x-1} < 2$

4. Solve simultaneously:
$$\begin{aligned} a + b + c &= 0 \\ a - b + c &= 6 \\ 4a - 2b + c &= 15 \end{aligned}$$

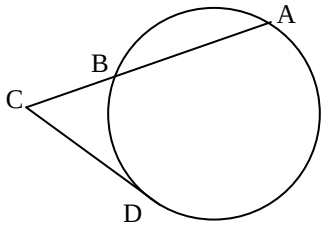
Question 2 — Circle Geometry

1. Find the values of x and y. Show reasons



2. Chord AB = 5.8 cm, CB = 3.1 cm and CD is a tangent to the circle.

Find the length of CD (correct to one decimal place).

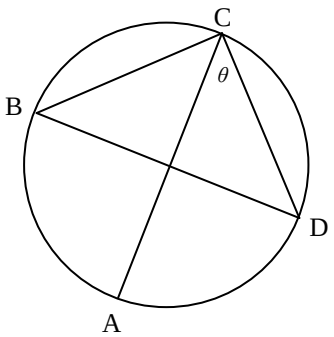


3. In the circle with centre O, $\angle OCD = \theta$,

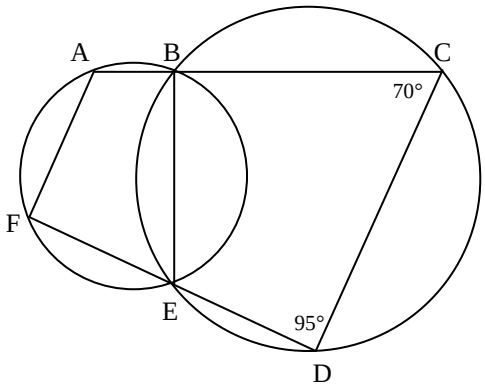
(a) find $\angle AOD$

(b) find $\angle BCO$

(c) show that arc AD = arc BC



4. In the intersecting circles,
$\angle BCD = 70°$ and $\angle CDE = 95°$.



(a) Find $\angle BAF$

(b) Find $\angle AFE$

(c) Hence, or otherwise, prove that $AF \| CD$

Question 3 **Functions and Graphs**

1. Find the value of x for which $f(x) = 8$ given that $f(x) = 3x - 10$

2.

$$g(x) = \begin{cases} 2x & \text{if } x \geq 3 \\ 1 & \text{if } -1 < x < 3 \\ x^3 - 4 & \text{if } x \leq -1 \end{cases}$$

Find the value of $g(5) - g(2) - g(-2)$.

4. Sketch the following, showing all relevant features:

(a) $y = |x| - 2$

(b) $y = x - x^2$

(c) $f(x) = -\sqrt{16 - x^2}$

(d) $g(x) = \frac{1}{x-4}$

(e) $y = -2^x$

(f) $y = \frac{1}{x^2}$

5. State the domain and range of the following:

(a) $y = |x|$

(b) $x^2 + y^2 = 9$

(c) $y = \sqrt{2x + 8}$

(d) $f(x) = x^2 - 7$

(e) $xy = -2$

6. Sketch the region defined by:

(a) $x < 5$, $2x + 3y < 6$

(b) $y \geq x^2$, $x < 2$

Question 4 **Locus and the Circle**

1. Find the locus of a point $P(x, y)$ that moves such that it is always equidistant from two fixed points, A(3, -1) and B(-4, 2).

2. Find the equation of the locus of the point $P(x, y)$ that moves so that the line PA is perpendicular to the line PB, where A is (1, 3) and B is (-2, 4).

3. Point $P(x, y)$ moves so that $PA^2 + PB^2 = 10$ where A and B are (2, 0) and (-2, 0) respectively. Find the equation of the locus and sketch it on a number plane.

4. Find the equation of the circle with centre (3, -4) and radius 5 units.

5. Find the centre and radius of the circle with equation $x^2 + y^2 + 4x + 2y + 1 = 0$.

Question 5 **Introductory Calculus**

1. Find: (a) $\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 4x + 3}$

(b) $\lim_{x \to \infty} \frac{2x^2 - x + 1}{x^2 + 3}$

2. (a) Given that $f(x) = x^2 + 3x$, find $f(x + h)$.

(b) Hence find $f'(x)$ from first principles.

3. Find the derivative of: (a) $3x^4 + 2x^3 - 4x + 5$

(b) $\sqrt[5]{x}$

(c) $\frac{3}{x^6}$

(d) $(x + 3)(2x - 1)$

(e) $(4x - 5)^7$

(f) $\frac{5x^3}{2x - 3}$

(g) $x^5(9x^2 + 7)^6$

4. Given that $g(x) = x^4 - 3x$, find the value of $g'(-2)$

5. Find the equation of the normal to the curve $f(x) = x^3 - 7x$ at the point (2, -6)

Question 6 **Trigonometry**

1. A ship sails from port (P) on a bearing of 200° for 82km. How far west of the port is the ship (to the nearest kilometre)?

2. The angle of depression from the top of a 350 m building to a car is 72°14′. How far is the car from the building (to 2 decimal places)?

3. A 53.7 m building casts a shadow 109.3 m long at a certain time of day. What is the angle of elevation that the shadow makes with the top of the building at the time (in degrees and minutes)?

4. Prove that $\cot\theta + \tan\theta = \text{cosec}\theta.\sec\theta$

5. Find the value of b if $\sin(20° + 2b) = \text{cosec}(b + 10°)$

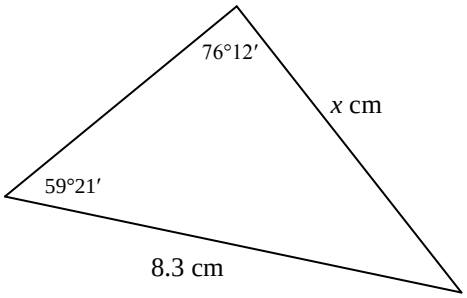
6. Find the exact value of: (a) cos150°

(b) tan300°

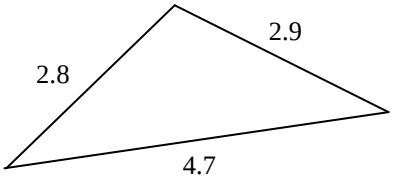
(c) tan(-135°)

7. If $\tan\theta = -\frac{3}{4}$ and $\cos\theta < 0$, find $\sin\theta$ as rational number.

8. Find the value of x correct to 3 significant figures.



9. Find the size of the largest angle to the nearest degree.



10. Two cars leave an intersection at the same time, one travelling at 60 km/h along one road and the other car travelling at 80 km/h along the other road. After 3 hours they are 315 km apart.

    (a) Draw a diagram to show this information,

    (b) At what angle do the roads meet at the intersection (to the nearest minute)?

11. Find the area of triangle OAB where O is the centre of the circle.

    Give your answer to the nearest mm^2.