DAPTO HIGH SCHOOL

2008
HSC Preliminary Course
FINAL EXAMINATION

Mathematics – Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Board approved calculators may be use
- All necessary working should be shown for every question
- Begin each question on a fresh sheet of paper

Total marks (87)

- Attempt Questions 1 – 6

Question 1 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) Solve the inequality $\frac{3x}{3x-4} \leq 3$ **3**

(b) Solve $a - b + c = 7$ and $a + 2b - c = -4$ and $3a - b - c = 3$ simultaneously. **3**

(c) Show that the line $x + y + 4 = 0$ is a tangent to the circle $x^2 + y^2 = 8$. **3**

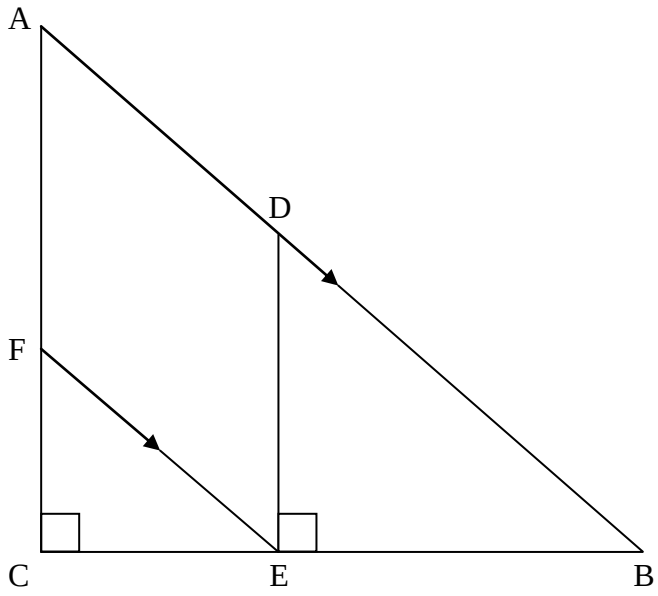
(d) Find the acute angle between the lines $x + 2y = 0$ and $x - 3y = 0$. **3**

(e) For the points A(–3, –7) and B(–1, –4). Find the coordinates of the point P(x , y) which divides the interval AB externally in the ratio 4:3. **3**

End of Question 1

Question 2 (12 Marks) Use a Separate Sheet of paper **Marks**

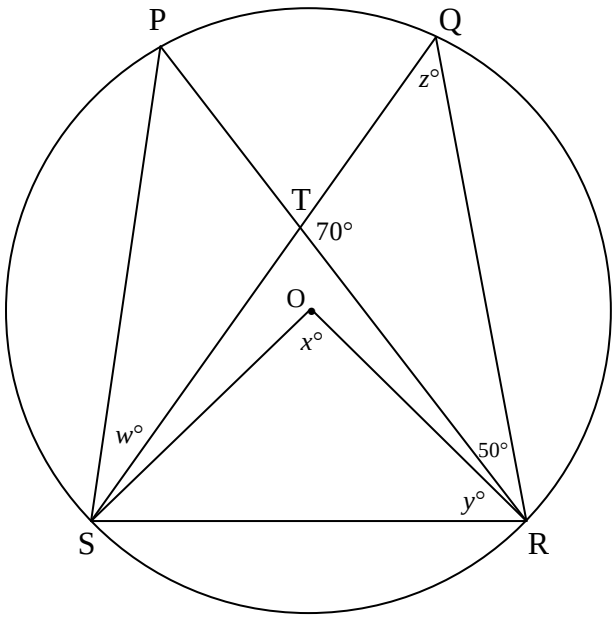
(a) In the triangle below, $AB \parallel FE$ and $\angle FCE = \angle DEB = 90°$. **4**



i) Prove that $\Delta ABC \,|||\, \Delta DBE$ and $\Delta ABC \,|||\, \Delta FCE$

ii) If $DE : FC = 5 : 2$, $FE = 3.2$cm and $CE = 2.4$cm, find the length of AB

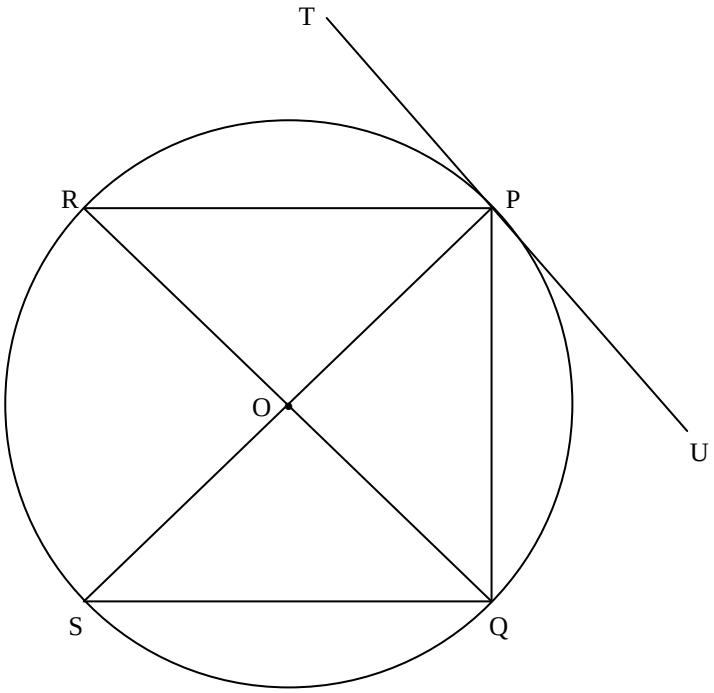
(b) Find the values of w, x, y and z, giving reasons. **4**



Question 2 continues on page 4

Question 2 (continued) **Marks**

(c) The point O is the centre of the circle, TU is a tangent to the circle, contacting the circle at P. **4**



i) Show that $\angle ROP = 2\angle RPT$

ii) Show that $\angle RPT$ and $\angle QPU$ are complementary

iii) Show that $RP \parallel SQ$

End of Question 2

Question 3 (15 Marks) Use a Separate Sheet of paper **Marks**

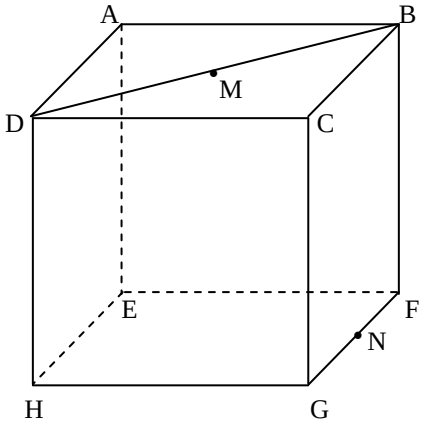
(a) Write $\sin\theta + \cos\theta$ in terms of t where $t = \tan\frac{\theta}{2}$. **2**

(b) The cube ABCDEFGH has sides of length 12cm. M is the midpoint of the diagonal BD on the face ABCD and N is the midpoint of the edge GF. **4**

i) Calculate the length of the line MH

ii) Find the size of $\angle MHF$ to the nearest minute

iii) Find the size of $\angle MHN$ to the nearest minute



(c) Show that $\sin(2x + 30°) = \sqrt{3}\sin x\cos x + \cos^2 x - \frac{1}{2}$. **2**

(d) Find "k" if $kx^2 - 7x + k + 1 = 0$ and one root is -2. **3**

(e) i) Express $\sqrt{2}\sin x + \cos x$ in the form $r\sin(x + \alpha)$. **2**

ii) Hence or otherwise, solve $\sqrt{2}\sin x + \cos x = 1$ for $0° \le x \le 360°$, giving your answer to the nearest minute.

End of Question 3

Question 4 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the centre and radius of the circle with equation $x^2 + 8x + y^2 - 12y + 27 = 0$. **3**

(b) Differentiate from first principles and find the gradient of the tangent at the curve $y = x^2 + 10$ at $x = 3$. **4**

(c) Find: **4**

i) $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$

ii) $\lim_{x \to \infty} \frac{3x^2}{x^2 - 2x + 3}$

(d) Find the equation of the locus at a point P which moves such that it's distance from the point A(2, –3) is twice it's distance from the point B (–4, 12). **4**

End of Question 4

Question 5 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) Express $5x^2 + 2x - 3$ in the form $A(x + 1)^2 + B(x + 1) + c$ **2**

(b) Solve $9^x - 10(3^x) + 9 = 0$ **2**

(c) Find values of k for which $x^2 + kx + 16$ is positive definite. **2**

(d) For the equation $2x^2 + x - 3 = 0$ with roots α and β, find the value of:

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ **1**

iv) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ **2**

(e) For the parabola defined by $x^2 - 8y = 0$, find the:

i) Co-ordinates of the vertex **1**

ii) Co-ordinates of the focus **1**

iii) Equation of the directrix **1**

iv) Sketch the parabola **1**

End of Question 5

Question 6 (15 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the derivative of the following: (You do not need to simplify your answers after finding the derivative)

i) $-4x^5 - 4x^3 + 11$ **1**

ii) $\sqrt[5]{x^2}$ **2**

iii) $\dfrac{1}{x+3}$ **2**

(b) i) Find $f'(2)$ for $f(x) = (3x^2 - 5x)^5$ **2**

ii) If $y = \sqrt{3x - 1}$ find $\dfrac{dy}{dx}$ **2**

iii) Given $y = \dfrac{x^2 - 1}{x^2 + 1}$ **2**

iv) Find $g'(x)$ if $g(x) = (2x + 3)^5 (5x - 1)$ **2**

(c) Find the gradient of the curve $y = x^2 - 2$ at $x = 4$ and hence find the equation of the tangent to this curve at $x = 4$. **2**

End of Question 6

End of Examination