Year 12 Extension 1 **Applications of Calculus to the Physical World (3U) Assignment** **Date Due:**

1. A spherical balloon is being inflated and its radius is increasing at the constant rate of 3 cm/min. At what rate is its volume increasing when the radius of the balloon is 5 cm?

2. The volume of water in a hemispherical bowl of radius 10 cm is given by $V = \frac{\pi}{3}x^2(30 - x)$ where x cm is the depth of the water at any time t. The bowl is being filled at a constant rate of 2π cm^3/min. At what rate is the depth increasing when the depth is 2 cm?

3. If the original temperature of a body is 100°C, the temperature of the surroundings is 20°C and the body cools to 70°C in 10 minutes, find, assuming Newton's law of cooling,
   (a) the temperature after 20 minutes.
   (b) the time taken to reach 60°C

4. A particle moves in a straight line so that any time t its displacement from a fixed origin is x and its velocity is v. If the acceleration is $3x^2$ and $v = -\sqrt{2}$, $x = 1$ when $t = 0$, find x as a function of t.

5. If $\frac{dx}{dt} = \frac{1}{x+4}$ and $x = 0$ when $t = 0$, find t when $x = 2$.

6. The equation of motion of a particle moving along a straight line is given by the equation
$$\frac{d^2x}{dt^2} = -16x$$
If initially the particle is at the origin moving with a velocity 4 m/s in the positive direction, find its displacement at any time t and state the period and amplitude.

7. The speed v (cm/s) of a particle moving in a straight line is given by $v^2 = 6+4x - 2x^2$ where the magnitude of its displacement from a fixed point 0 is x (cm).

   Show that the motion is simple harmonic and find:
   (a) the centre of the motion
   (b) the period
   (c) the amplitude

8. A particle moves in a straight line and its position at any time t is given by $x = 3\cos 2t + 4 \sin 2t$
   Show that the motion is simple harmonic and find its greatest speed.

9. A particle is projected from a horizontal plane at an angle of elevation of 30° with a speed of 100 metres/second. Find:
   (a) the equation of the trajectory
   (b) the range of the projectile and the time of flight.
   (c) the maximum height.

10. A particle is projected from a window 9 metres above the horizontal ground at an angle of α to the horizontal where $\tan\alpha = \frac{3}{4}$ with an initial velocity of 20 metres/second. Neglecting air resistance, find:
    (a) the Cartesian equation of the path,
    (b) the maximum height above the ground,
    (c) the time taken for the particle to strike the ground,
    (d) the horizontal distance from the window to the point where it strikes the ground.