Applications of Calculus to the Physical World

- **Velocity as a Derivative**
  The velocity of a particle, moving in a straight line, so that its position is x at time t, then velocity is the derivate of the position / equation. Can be written as $\dot{x}$

- **Acceleration as Derivative**
  The acceleration is the derivative of the velocity or the 2^{nd} derivative of the original equation. Can be written as $\ddot{x}$

- **Velocity and Acceleration Involving Integration**
  If a particle moves along a straight line so that at time t, its position is x, its velocity is v and its acceleration is a then:
  $v = \frac{dx}{dt}$ and thus $x = \int v\,dt$

  $a = \frac{dv}{dt}$ and thus $v = \int a\,dt$

- **Derivation of Formulae for Constant Acceleration**
  $v = u + at$
  $s = ut + \frac{1}{2}at^2$
  $v^2 = u^2 + 2as$

  where:
  a is constant acceleration
  u is initial velocity
  t is time
  v is velocity
  s is change in displacement

- **Velocity as a function of position**
  Velocity is a function of x, with velocity; we can find t by reciprocating.

- **Acceleration as a function of position**
  - Acceleration as a function of x
  - for a moving particle, if the position and velocity at time t are x and v respectively then:
  $\frac{d^2x}{dt^2} = \frac{d}{dx}(1/2\ v^2) = v\frac{dv}{dx} = a$

- **Parabolic Motion Under Gravity: Projectiles**
  For a projectile with initial velocity v in the direction making an angle θ with the horizontal, then for motion:
  In the x-direction: $\ddot{x} = 0$, $\dot{x} = V\cos\theta$, $x = Vt\cos\theta$
  In the y-direction: $\ddot{y} = -g$, $\dot{y} = -gt + V\sin\theta$, $y = -\frac{1}{2}gt^2 + V\sin\theta$,

- **Simple Harmonic Motion**
  For the particle P moving in SHM $(-n^2x)$ and assuming the conditions when t=0, v=0 and x=a then:
  $v^2 = n^2(a^2 - x^2)$ and $x = a\cos nt$
  If x = a cost nt, then $v = -an\sin nt$ and $f = -an^2\cos nt$ if $\ddot{x} = -n^2x$, then the period is $T = \frac{2\pi}{n}$

- **Alternate treatment for SHM**
  A point moving in a straight line is describing SHM if: $x = a\cos(nt + \alpha)$ where a , n and α are constants and $a>0$ and $n>0$.
  $\dot{x}^2 = v^2 = n^2(a^2 - x^2)$ and $\ddot{x} = -n^2x$
  period is $T = \frac{2\pi}{n}$, frequency is $\frac{n}{2\pi}$