Area Under the Curve

- **Fundamental Theorem of Calculus**
  $\int_a^b F(x)dx = F(b) - F(a)$, where $F(x)$ is the primitive function of $f(x)$

- **Definite and Indefinite Integrals**
  - $\int x^2$ dx is indefinite as it has no physical significance
  - $\int_2^5 x^2 dx$ is definite as it is bounded by $y = x^2$, $x = 2$, and $x = 5$

- **A Special Result**
  $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n + 1)} + C$

- **Area bounded by the x axis**
  $\int_a^b F(x)dx = F(b) - F(a)$, where $F(x)$ is the primitive function of $f(x)$

- **Area bounded by the y axis**
  - first make the function in terms of *x*
  - it then becomes the same process are bounded by the y axis
  - $\int_c^d g(y)dx = F(d) - F(e)$, where $G(y)$ is the primitive function of $g(y)$

- **Area below the x axis**
  - We have to take the absolute value as being under the *x*-axis gives a negative area
  - $\left|\int_a^b f(x)dx\right|$

- **Volumes of solids of revolution**
  - If we rotate the area bounded by the curve $y = f(x)$, by the x axis and the lines $x = a$ and $x = b$:
  - $V= \int_a^b \pi y^2 \, dx = \pi \int_a^b [f(x)]^2$
  - If by the y axis, then the volume is:
  - $V= \int_c^d \pi x^2 \, dy = \pi \int_c^d [g(y)]^2 \, dy$

- **The Trapezoidal Rule**
  - $\int_a^b f(x)dx \approx \frac{h}{2}\left[(y_0+y_n)+ 2(y_1+y_2+y_3+ \ldots + y_{n-1})\right]$
  - where $h = \frac{b - a}{n}$
  - *ie.* $A = \frac{h}{2}$[(sum of end ordinates)+ 2(sum all remaining {middle} ordinates)]

- **Simpson's Rule**
  - $\int f(x)dx=\frac{h}{3}\{(y_0+ y_n) + 4(y_1+ y_3+ y_5+ \ldots) + 2(y_2+ y_4 + \ldots)$
  - where $h = \frac{b - a}{n}$
  - *ie.* $A = \frac{h}{3}$[(sum of end ordinates)+ 4(sum of odd ordinates) + 2(sum of even ordinates)]