Year 12 Extension 2 **Complex Numbers Assignment** **Date Due:**

1. Solve the following equations:
   (a) $x^2 + 16 = 0$ (b) $x^2 + x + 1$ (c) $x^2 + 5 = 0$
   (d) $-x^2 + 2x - 5 = 0$ (e) $x^2 = 6x - 20$ (e) $-2x^2 + 2x - 13 = 0$
2. Given that $z = 2 + 3i$ and $w = 3 + i$, find:
   (a) $z + w$ (b) $z - w$ (c) zw (d) $\overline{w}$ (e) $\frac{w}{z}$ (f) $\frac{z}{w}$
   (g) $z\overline{z}$
3. Simplify: $\frac{5i}{5+6i} + \frac{3}{2-i}$
4. Represent each of the following on the complex number plane:
   (a) $2 + i$ (b) $3 + 4i$ (c) $7 - 2i$ (d) $-i$ (e) $2 - 2i$
5. If $z_1 = 5 + 4i$ and $z_2 = 4 + 2i$, find each of the following and then represent on a number plane:
   (a) $z_1 + z_2$ (b) $z_1 - z_2$
6. Express each of the following in modulus-argument form:
   (a) $1 - i$ (b) $-3 + 3i$ (c) $4 - 3i$ (d) i (e) 3
7. Express each of the following in Cartesian form:
   (a) $3\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ (b) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ (c) $5\left(\cos-\frac{\pi}{6} + i\sin-\frac{\pi}{6}\right)$
   (d) $2\left(\cos-\frac{2\pi}{3} + i\sin-\frac{2\pi}{3}\right)$ (e) $5\text{cis}\frac{\pi}{3}$ (f) $3\text{cis}-\frac{\pi}{4}$
8. Find an expression for the following in Cartesian form:
   (a) $2\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \times 5\left(\cos\frac{\pi}{4} + i\sin-\frac{3\pi}{4}\right)$ (b) $5\text{cis}\frac{\pi}{3} \times 3\text{cis}\frac{3\pi}{4}$
   (c) $\dfrac{\sqrt{3}\text{cis}\frac{3\pi}{4}}{\sqrt{3}\text{cis}\frac{\pi}{4}}$ (d) $\frac{2 - 2i}{-1+i}$ (e) $-2i \times \left(\sqrt{5} - i\right)$
9. Express each of the following in Cartesian form:
   (a) $\left(2\text{cis}\frac{2\pi}{3}\right)^6$ (b) $\left(\sqrt{3} - i\right)^5$ (c) $(-2 + 2i)^4$ (d) $\left[2\left(\cos\frac{7\pi}{10} + i\sin\frac{7\pi}{3}\right)\right]^5$
   (e) $[3(\cos 5° + i\sin 5°)]^6$ (f) $\left(\sqrt{2} - i\right)^{-4}$ (g) $\left(-4 - 2\sqrt{3}i\right)^{-3}$
10. Simplify: $\dfrac{\left[3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)\right]^6}{\left[2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\right]^2}$
11. Find the values of z for where $z = x + yi$ if:
    (a) $z^6 = 1$ (b) $z^4 = -16$ (c) $z^3 + 6i = 0$ (d) $z^3 = \frac{\sqrt{3} - i}{2}$
    (e) $z^2 = 6\text{cis}\frac{\pi}{2}$ (f) $z^6 = 27\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$
12. If ω is the complex root of $z^6 - 1$:
    (a) Prove that ω, ω^2, ω^4 and ω^5 are the roots of $z^4 + z^2 = 0$
    (b) Find the quadratic equation whose roots are $\alpha = \omega + \omega^5$ and $\beta = \omega^2 + \omega^4$.
13. Verify the triangle inequalities for:
    (a) $z_1 = 3 - 7i$, $z_2 = -2 + 5i$
    (b) $z_1 = 3 + 2i$, $z_2 = -2 + 3i$