

## Complex Numbers

- **The number  $i$**

$$i^2 = -1$$

- **The Quadratic Formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- **Properties of the Complex Number System**

- When adding (or subtracting) complex numbers add (or subtract) the real and imaginary parts separately
- Eg.  $(a + bi) + (c + di) = (a + c) + (b + d)i$
- $\bar{z}$  is the conjugate of  $z$
- Eg. if  $z = (a + bi)$  then  $\bar{z} = (a - bi)$
- The product of a pair of conjugates is a real number
- Eg.  $(x + yi)(x - yi) = x^2 + y^2$
- To realise the denominator, multiply by the conjugate

- **The Complex Number Plane (Argand diagram)**

- Horizontal axis represents  $\operatorname{Re}(z) = x$  and Vertical axis represents  $\operatorname{Im}(z) = y$
- The addition (of subtraction) of two complex numbers forms a parallelogram

- **Multiplication of a Complex Number by  $i$**

- Equivalent to a  $90^\circ$  multiplication in the anti clockwise direction

- **Polar Form of a complex number (Modulus-Argument Form)**

- $z = x + yi = r(\cos\theta + i\sin\theta) = r\operatorname{cis}\theta$
- $|z| = |x + yi| = \sqrt{x^2 + y^2} = r$
- $\arg(z) = \tan \theta = \frac{y}{x}$

- **Multiplication and Division in Modulus-Argument Form**

- $z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$
- $\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$

- **De Moivre's Theorem**

- $z^n = r^n \operatorname{cis} n\theta = r^n (\cos n\theta + i \sin n\theta)$

- **Roots of Complex Numbers**

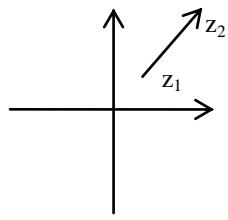
- for  $z = r(\cos\theta + i\sin\theta)$ :
- $|z|^{\frac{1}{n}} = r^{\frac{1}{n}}$
- $\operatorname{Arg}(z) = \frac{\theta + 2k\pi}{n}$  where  $k = 0, \pm 1, \pm 2, \dots$

- **Theorems on the Modulus and Argument**

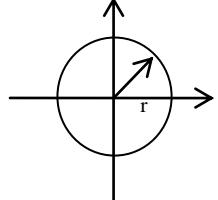
- $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$  and  $\arg(z_1 \cdot z_2) = \arg z_1 + \arg z_2$
- $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$  and  $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

- **Geometric Illustrations of Complex Numbers**

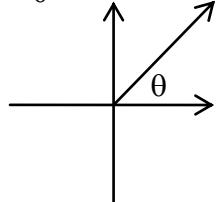
- $z_1 - z_2$



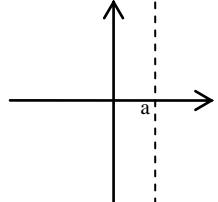
- $|z| = r$



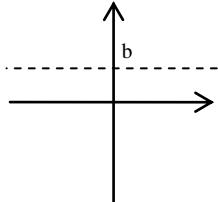
- $\operatorname{Arg}(z) = \theta$



- $\operatorname{Re}(z) = a$



- $\operatorname{Im}(z) = b$



- **Properties of Conjugates**

- $|z| = |\bar{z}| = \sqrt{x^2 + y^2}$

- $\arg z = -\arg z$