

CONICS

RECTANGULAR HYPERBOLA

1. Find the parametric coordinates of a point on each of the following curves:
(a) $xy = 9$ (b) $xy = 16$
(c) $4xy = 25$ (d) $xy = 1$
(e) $xy = 2$ (f) $xy = -4$
2. For the following find the Cartesian equation and the coordinates of the vertices:
(a) $(5t, \frac{5}{t})$
(b) $(3t, \frac{3}{t})$
(c) $(6t, \frac{6}{t})$
(d) $(t, -\frac{1}{t})$
3. Sketch the following rectangular hyperbola
(a) $2x = \frac{25}{y}$
(b) $x = 3t, y = \frac{3}{t}$
(c) $x = 4t, y = \frac{4}{t}$
(d) $xy = -2$
4. Find the coordinates of the foci of the following curves
(a) $xy = 18$
(b) $xy = 4$
(c) $x = 8t, y = \frac{8}{t}$
(b) $2xy = 25$
5. Find the equation of the tangents at the vertices of the rectangular hyperbola $xy = 8$.
6. Find the equation of the tangent and the normal to the curve $xy = 16$ at the point $(4t, \frac{4}{t})$.
7. Find the equations of the tangents to the curve $xy = 3$ which are parallel to the line $y + 3x = 0$. What is the distance between the tangents?
8. Find the points of contact of the two tangents which can be drawn from the point $(-5, 1)$ to the curve $xy = 4$.
9. The normal at the point $P(8, 2)$ on the curve $x = 4t, y = \frac{4}{t}$ meets the curve again at Q . Find the length of PQ .
10. Find the equations of the tangents to the curve $xy = 9$ which pass through the point $(-9, 3)$.
11. Find the equation of the tangent at the point $P(6, 2)$ of the rectangular hyperbola $xy = 12$. If this tangent meets the coordinate axes in Q and R , show that $QP = PR$.
12. (a) Show that if $y = mk + k$ is a tangent to the rectangular hyperbola $xy = c^2$, then $k^2 + 4mc^2 = 0$.
(b) Hence find the equations of the tangents from the point $(-1, -3)$ to the rectangular hyperbola $xy = 4$ and find the coordinates of their points of contact.

13. $P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ lie on the rectangular hyperbola $xy = c^2$. The chord PQ subtends a right angle at another point $R(cr, \frac{c}{r})$ on the hyperbola. Show that the normal at R is parallel to PQ.
14. $P(ct, \frac{c}{t})$, where $t \neq 1$, lies on the rectangular hyperbola $xy = c^2$. This tangent and normal at P meet the line $y = x$ at T and N respectively. Show that:
 (a) $OP = PN$ (b) $OT \cdot ON = 4c^2$
15. $P(cp, \frac{c}{p})$ and $Q(cq, \frac{c}{q})$ lie on the rectangular hyperbola $xy = c^2$. The tangents at P and Q meet at R, and OR cuts PQ at M. Show that M is the midpoint of PQ.
16. P and Q are variable points on the rectangular hyperbola $xy = 9$. The tangents at P and Q meet at R. If PQ passes through the point (6, 2), find the equation of the locus of R.
17. P and Q are variable points on the rectangular hyperbola $xy = c^2$. The tangents at P and Q meet at R. If PQ passes through the point (a, 0), find the equation of the locus of R.
18. The following questions all refer to the rectangular hyperbola $(cp, \frac{c}{p})$
 (a) The line $2x + y = 3c$ meets the asymptotes in U, V. Prove that the hyperbola passes through the two points of trisection UV
 (b) P, Q are two variable points on the hyperbola $xy = c^2$, such that the tangent at Q passes through the foot of the ordinate P. Show that the locus of the midpoint of PQ is a hyperbola with the same asymptotes as the given hyperbola.
19. P is a variable point on the rectangular hyperbola. The tangent at P cuts the x axis and the y axis at A and B respectively. Q is the fourth vertex of the rectangle whose other vertices are the origin, A and B. Show that as P moves along the circle $xy = c^2$, Q also describes a hyperbola with the same asymptotes.
20. The tangents at P and Q on the rectangular hyperbola meet at R. Show that the locus of R is a straight line through O if P and Q moves on the curve with the equation $xy = c^2$, with parameters p and q respectively and if $pq = k$ (constant).
21. The tangent at $P(x_1, y_1)$, a variable point on the hyperbola $x^2 - y^2 = a^2$, meets the asymptotes $y = x$ at Q. Show that the locus of M(X, Y), the midpoint of PQ, has equation:

$$X^2 - Y^2 = \frac{3a^2}{4}$$
22. The normal to the rectangular hyperbola $x^2 - y^2 = 1$ at P meets the asymptotes at G and H. Show that the locus of M(X, Y), the midpoint of GH is

$$4X^2Y^2 = (Y^2 - X^2)^3$$