CURVE SKETCHING

EXERCISE 1

Sketch the following

1. $y = x^2 - 4x + 3$
2. $y = -x^2 + 2x$
3. $y = x^2 + x + 1$
4. $x = y^{2\,+}\, 2y$
5. $y = (x - 1)^3$
6. $y = -x^3$
7. $y = (x + 2)(x - 1)(x - 3)$
8. $y = (x + 2)(x - 2)^2$
9. $y = (x + 2)(x + 1)(x - 1)(x - 3)$
10. $y = |x - 2|$
11. $xy = 2$
12. $x^2 + y^2 = 4$
13. $x^2 + y^2 + 8x - 4y + 11 = 0$
14. $y = 3^x$
15. $y = \log_4 x$
16. $y = \dfrac{x^2}{(x + 2)(x - 1)}$
17. $y = \dfrac{(x + 1)(x - 1)}{(x + 2)(x - 2)}$
18. $y = \dfrac{x(x + 2)}{x^2 - 1}$

IMPLICIT DIFFERENTIATION

1. Differentiate with respect to x where y is a function of x

   (a) x^3y (b) $\frac{x}{y}$ (c) $\frac{x^n}{y^n}$

2. Find $\frac{dy}{dx}$ in each of the following.

   (a) $x^3 + y^2 = a$ (b) $\sqrt{x} - \sqrt{y} = b$

   (c) $x^4 + 5ax^2y - 6ay^2 + 3x = 2$ (d) $x^p y^q = pq$

   (e) $(x^3 + y^3)^2 = 2(x^3 - y^3)$ (f) $ax^2 + 2bxy + cy^2 + dy + ex + f = 0$

3. Find $\frac{dv}{dp}$ in each of the following equations.

   (a) $pv = c$ (b) $pv^2 = c$ (c) $(p + \frac{d}{v^2})(v - b) = c$

4. Find the gradient of the tangent to the curve $\frac{x^2}{2} + \frac{xy}{2} + y^2 = 14$ at (2, 3). Hence write the equation of this tangent.

5. Show that the curves $y^2 = 4ax + 4a^2$ and $y^2 = 4a^2 - 4ax$ intersect at right angles.

6. If $x^2y = (x + y)^3$, show that $\frac{dy}{dx} = \frac{y}{x}$

7. If $x^2 - y^2 = 10$, show that $\frac{dv}{dx} = \frac{y}{x}$ and $\frac{d^2v}{dx^2} = \frac{-10}{y^3}$

1. For the function $f(x) = x^2$ (an even function) sketch the graphs of:
   (a) $y = f(x)$ (b) $y = f(-x)$ (c) $y = -f(x)$

2. For the function $f(x) = x^3$ (an odd function) sketch the graphs of:
   (a) $y = f(x)$ (b) $y = f(-x)$ (c) $y = -f(x)$

3. Use the graph of $y = \ln x$ to sketch the graphs of
   (a) $y = \ln(-x)$ (b) $y = -\ln x$

4. Use the graph of $y = \ln x$ to sketch the graphs of
   (a) $y = |\ln x|$ (b) $y = \ln |x|$

5. Use the graph of $f(x) = 4 - x^2$ (an even function) to sketch (showing critical points) the graph of $y = |f(x)|$. Is this the graph an even function?

6. Use the graph of $f(x) = x^3 - 3x$ (an odd function) to sketch (showing critical points) the graph of $y = |f(x)|$. Is this the graph an even function?