Solutions for Differentiation Assignment.

1. $y = x^3 + 3x + 6x^2 + 18$

$\frac{dy}{dx} = 3x^2 + 3 + 12x$

2. $y = 2x^4 - 10x^2$

$\frac{dy}{dx} = 8x^3 - 20x$

Expand rather than use product rule

3. $\frac{dy}{dx} = 4(3x+4)^3 . 3$

$= 12(3x+4)^3$

4. $y = (2x+8)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(2x+8)^{-\frac{1}{2}} . 2$

$= \frac{1}{\sqrt{2x+8}}$

5. $y = (x+1)^{-1}$

$\frac{dy}{dx} = -1(x+1)^{-2} . 1$

$= \frac{-1}{(x+1)^2}$

Use chain rule

6. $\frac{dy}{dx} = \frac{(2x-1).2 - (2x+1).2}{(2x-1)^2}$

$= \frac{4x-2-4x-2}{(2x-1)^2}$

$= \frac{-4}{(2x-1)^2}$

7. $\frac{dy}{dx} = \frac{(1-x^2).2x - x^2.-2x}{(1-x^2)^2}$

$= \frac{2x - 2x^3 + 2x^3}{(1-x^2)^2}$

$= \frac{2x}{(1-x^2)^2}$

Use quotient rule

8. $y = (2x+3)^{-5}$

$\frac{dy}{dx} = -5(2x+3)^{-6} . 2$

$= \frac{-10}{(2x+3)^6}$

chain rule

9. $\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}}$

$= \frac{2}{3\sqrt[3]{x}}$

10. $y = x^4 - 6x^2 + x^2 - 6$

$y = x^4 - 5x^2 - 6$

$\frac{dy}{dx} = 4x^3 - 10x$

expand rather than use product rule.

11. $y = (4x-1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(4x-1)^{-\frac{1}{2}} . 4$

$= \frac{2}{\sqrt{4x-1}}$

chain rule

12. $\frac{dy}{dx} = \frac{(x+1).1 - x.1}{(x+1)^2}$

$= \frac{1}{(x+1)^2}$

quotient rule

13. $\frac{dy}{dx} = 4(3-2x)^3 . -2$

$= -8(3-2x)^3$

14. $y = (2-3x)^{-4}$

$\frac{dy}{dx} = -4(2-3x)^{-5} . -3$

$= \frac{12}{(2-3x)^5}$

chain rule

15. $y = x^{\frac{3}{2}} + x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{3\sqrt{x}}{2} + \frac{1}{2\sqrt{x}}$

$= \frac{3x+1}{2\sqrt{x}}$

expand & differentiate each term

16. $\frac{dy}{dx} = x^3 . 3(1+x)^2 + (1+x)^3 . 3x^2$

$= 3x^2(1+x)^2(x+1+x)$

$= 3x^2(1+x)^2(2x+1)$

product rule & factorise

17. $\frac{dy}{dx} = \frac{(3x+5).5 - (5x-2).3}{(3x+5)^2}$

$= \frac{15x+25-15x+6}{(3x+5)^2}$

$= \frac{31}{(3x+5)^2}$

18. $\frac{dy}{dx} = \frac{(x+1).3x^2 - (x^3-1).1}{(x+1)^2}$

$= \frac{3x^3 + 3x^2 - x^3 + 1}{(x+1)^2}$

$= \frac{2x^3 + 3x^2 + 1}{(x+1)^2}$

quotient rule.

19. $y = \dfrac{x}{(x^2+1)^{\frac{1}{2}}}$

$$\frac{dy}{dx} = \frac{(x^2+1)^{\frac{1}{2}}.1 - x.\frac{1}{\cancel{2}}(x^2+1)^{-\frac{1}{2}}.\cancel{2}x}{x^2+1}$$

$$= \frac{\sqrt{x^2+1} - \dfrac{x^2}{\sqrt{x^2+1}}}{x^2+1}$$ (quotient rule)

$$= \frac{\dfrac{(x^2+1) - x^2}{\sqrt{x^2+1}}}{x^2+1}$$

$$= \frac{1}{(x^2+1)^{\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{(x^2+1)^3}}$$

20. $\dfrac{dy}{dx} = (x+1)^2.3\,(x+2)^2.1 + (x+2)^3.2\,(x+1).1$ (product rule)

$= (x+1)\,(x+2)^2(3x+3+2x+4)$ factorise

$= (x+1)(x+2)^2(5x+7)$ simplify

21. $y = 2(x+1)^{-1}$

$\dfrac{dy}{dx} = -2(x+1)^{-2}.1$ ← use chain rule

$= \dfrac{-2}{(x+1)^2}$

At $x = 3$, $\dfrac{dy}{dx} = \dfrac{-2}{16} = \dfrac{-1}{8}$

$y = \dfrac{2}{4} = \dfrac{1}{2}$

$y - \frac{1}{2} = -\frac{1}{8}(x-3)$

$8y - 4 = -x + 3$

$x + 8y - 7 = 0$ is eqn of tangent

22. $y = \dfrac{x}{x^2+1}$

$\dfrac{dy}{dx} = \dfrac{(x^2+1)1 - x.2x}{(x^2+1)^2}$ ← quotient rule

$= \dfrac{x^2+1-2x^2}{(x^2+1)^2}$

$= \dfrac{1-x^2}{(x^2+1)^2}$

At $x = 0$, $\dfrac{dy}{dx} = \dfrac{1}{1} = 1$

$y - 0 = 1(x-0)$

$y = x$ is eqn of tangent

gradient of normal $= -1$

$y - 0 = -1(x-0)$

$y = -x$ is eqn of normal

23. Let $y = 0$

$(2x-5)^3 = 0$

$x = 2\frac{1}{2}$

$\dfrac{dy}{dx} = 3(2x-5)^2.2 = 6(2x-5)^2$

At $x = 2\frac{1}{2}$, $\dfrac{dy}{dx} = 0$

Slope of tangent $= 0$

24. $\dfrac{dy}{dx} = \dfrac{(4-x)2x - x^2.-1}{(4-x)^2}$ ← quotient rule

$= \dfrac{8x-2x^2+x^2}{(4-x)^2}$

$= \dfrac{8x-x^2}{(4-x)^2}$

At $x = 2$, $\dfrac{dy}{dx} = \dfrac{12}{4} = 3$

$y - 2 = 3(x-2)$

$3x - y - 4 = 0$ is eqn of tangent

25. a) $f(x) = \dfrac{x}{x^{\frac{1}{2}}} = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

b) $f'(x) = (2x+1)^3.3(5x-4)^2.5 + (5x-4)^3.2(2x+1).2$ (product rule)

$= (2x+1)(5x-4)^2(30x+15+20x-16)$

$= (2x+1)(5x-4)^2(50x-1)$

c) $$f'(x) = \frac{\sqrt{2x}.4 - 4x.\frac{1}{\cancel{2}}(2x)^{-\frac{1}{2}}.\cancel{2}}{2x}$$

$$= \frac{4\sqrt{2x} - \dfrac{4x}{\sqrt{2x}}}{2x}$$

$$= \frac{\dfrac{8x - 4x}{\sqrt{2x}}}{2x} = \frac{\overset{2}{\cancel{4x}}}{\sqrt{2x}} \times \frac{1}{\cancel{2x}}$$

$$= \frac{2}{\sqrt{2x}}$$

26. $y = x^{\frac{3}{2}}$

$\dfrac{dy}{dx} = \dfrac{3}{2}x^{\frac{1}{2}} = \dfrac{3\sqrt{x}}{2}$

At $x = 4$, $\dfrac{dy}{dx} = 3$

$y - 8 = 3(x-4)$

$3x - y - 4 = 0$ is eqn of tangent

grad of normal $= -\frac{1}{3}$

$y - 8 = -\frac{1}{3}(x-4)$

$3y - 24 = -x + 4$

$x + 3y - 28 = 0$ is eqn normal

At T, $x = \frac{\cancel{28}+4}{3}$ At N $x = 28$

$\therefore TN = 28 - 1\frac{1}{3} = 26\frac{2}{3}$