Differentiation

- **Differentiation from first principles**
  $\lim\limits_{x \to h} \dfrac{f(x+h) - f(x)}{h}$
  -or-
  $\lim\limits_{x \to c} \dfrac{f(x) - f(c)}{x - c}$

- **Using the alternate definition**
  $x^n - c^n = (x - c)(x^{n-1} + x^{n-2}c + \ldots + xc^{n-2} + c^{n-1})$

- **Basic rule for differentiation**
  if $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$

- **Function of a Function rule (chain rule)**
  - $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
  - if $y = [f(x)]^n$ then $\frac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$

- **The product rule**
  - if $y = u \cdot v$ where u and v are functions of x, then $\frac{dy}{dx} = u\frac{du}{dx} + v\frac{du}{dx}$
  - *ie. first × derivative of the second + second × derivative of the first*

- **The quotient rule**
  - if $y = \frac{u}{v}$ and u and v are functions of x, then $\frac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
  - *ie. bottom × derivative of the top – top × derivative of the bottom all over bottom squared*

- **The equation of a tangent to a point on a curve**
  - find the derivative of the curve
  - substitute the x value of the point into the derivate to find the gradient
  - find the y value if not given by substitution into the original curve
  - substitute the gradient, x and y values into the formula $y - y_1 = m(x - x_1)$ to find the equation of the tangent

- **The equation of a normal to a point on a curve**
  - find the derivative of the curve
  - substitute the x value of the point into the derivate to find the gradient
  - the gradient of the normal is the negative reciprocal
  - find the y value if not given by substitution into the original curve
  - substitute the gradient, x and y values into the formula $y - y_1 = m(x - x_1)$ to find the equation of the normal

- **Angle between two curves**
  $\tan\theta = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$