

Exponential & Logarithmic Functions

- **Derivatives of Exponential Functions**

- $\frac{d}{dx} [a^x] = r e^x$
- $\frac{d}{dx} [e^x] = e^x$
- $\frac{d}{dx} [e^{f(x)}] = f'(x) e^{f(x)}$

- **Integration of Exponential Functions**

- $\int e^x = e^x + C$
- $\int e^{ax+b} = \frac{1}{a} e^{ax+b} + C$
- $\int f'(x)e^{f(x)} dx = e^{f(x)} + C$

- **The Logarithmic Function**

- If $N = a^x$ then $\log_a N = x$
- $\log_a x + \log_a y = \log_a(xy)$
- $\log_a x - \log_a y = \log_a\left(\frac{x}{y}\right)$
- $\log_a x^n = n \log_a x$

- **Change of Base Result**

$$\log_a x = \frac{\log_b x}{\log_b a}$$

- **Derivative of $y = \log_e x$**

- if $y = \log_e x$ then $\frac{dy}{dx} = \frac{1}{x}$
- if $y = \log_e [f(x)]$ then $\frac{dy}{dx} = \frac{f'(x)}{f(x)}$

- **Integration of Logarithmic Functions**

- $\int \frac{1}{x} dx = \int x^{-1} dx = \ln x + C$
- $\int \frac{dx}{ax+b} = \frac{1}{a} \ln(ax+b) + C$
- $\int \frac{f'(x)}{f(x)} dx = \log_e f(x) + C$

- **Further Exponential and Logarithmic Results**

- $\frac{d}{dx} [a^x] = a^x \ln a$
- $\int a^x dx = \frac{1}{\ln a} a^x + C$
- $e^{\log_e x} = x$