EXTENSION 1 REVISION OF FORMULAE AND RESULTS

Co-ordinate Geometry

- Dividing an interval in the ratio $m:n$

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

- Acute angle between two lines (or tangents)

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

Trigonometric Ratios

- Sum and Difference Results

$$\sin(A+B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A-B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A+B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A-B) = \cos A\cos B + \sin A\sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

- Double Angle Results

$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$\cos 2A = 1 - 2\sin^2 A$$
$$\cos 2A = 2\cos^2 A - 1$$
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

- The 't' Formulae where $t = \tan\frac{\theta}{2}$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

- Subsidiary Angle Method ($R\sin(\theta + \alpha)$)

  When solving $a\sin\theta + b\cos\theta = c$ we can solve by writing in the form $R\sin(\theta + \alpha) = c$ where:

$$R = \sqrt{a^2 + b^2} \quad \text{and} \quad \tan\alpha = \frac{b}{a}$$

Parameters

- The parametric equations for the parabola $x^2 = 4ay$ are $x = 2at$ and $y = at^2$

- All other formulae in this subject are not to be committed to memory but students must know how they are derived.

Polynomials

- A real polynomial is in the form:

$$P(x) = p_n x^2 + p_{n-1}x^{n-1} + \ldots p_2 x^2 + p_1 x + p_0$$

- $p_1, p_2, p_3, \ldots.., p_n$ are *coefficients* and are real numbers, usually integers.

- The *degree* of the polynomial is the highest power of x with non-zero coefficient.

- A polynomial of degree n has at most n real roots but may have less.

- The result of a long division can be written in the form $P(x) = A(x) \,.\, Q(x) + R(x)$

- The *remainder theorem* states that when $P(x)$ is divided by $(x - a)$ the remainder is $P(a)$.

- The *factor theorem* states that if $x = a$ is a factor of $P(x)$ then $P(a) = 0$.

- If $\alpha, \beta, \gamma, \delta, \ldots$ are the roots of a polynomial then

$$\Sigma\alpha = -\frac{b}{a}, \ \Sigma\alpha\beta = \frac{c}{a}, \ \Sigma\alpha\beta\gamma = -\frac{d}{a}, \ \Sigma\alpha\beta\gamma\delta = \frac{e}{a}$$

Numerical Estimation of the Roots of an Equation

- Halving the Interval Method
- Newton's Method

  If $x = x_0$ is an approximation to a root of $P(x) = 0$ then $x_1 = x_0 - \frac{P(x_0)}{P'(x_0)}$ is generally a better approximation.

  Be familiar with the conditions under which this method fails.

Mathematical Induction

- Step 1: Prove result true for $n = 1$ (It is sometimes necessary to have a different first step.)
- Step 2: Assume it is true for $n = k$ and then prove true for $n = k + 1$
- Step 3: Conclusion as given in class

Integration

- $\int \sin^2\theta \, d\theta$ and $\int \cos^2\theta \, d\theta$ can be solved using the substitutions:

  $$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

  $$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$$

- Integration by first making a substitution.
- Table of Standard Integrals as provided in HSC

Inverse Trigonometric Functions

- $y = \sin^{-1}x$ Domain: $-1 \le x \le 1$ Range: $-\frac{\pi}{2} \le y \le -\frac{\pi}{2}$

  $y = \cos^{-1}x$ Domain: $-1 \le x \le 1$ Range: $0 \le y \le \pi$

  $y = \tan^{-1}x$ Domain: all real x Range: $-\frac{\pi}{2} \le y \le -\frac{\pi}{2}$

- Properties:

  $\sin^{-1}(-x) = -\sin^{-1}x$
  $\cos^{-1}(-x) = \pi - \cos^{-1}x$
  $\tan^{-1}(-x) = -\tan^{-1}x$
  $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$
  $\sin(\sin^{-1}x) = x$
  $\cos(\cos^{-1}x) = x$
  $\tan(\tan^{-1}x) = x$

- General Solutions of Trigonometric Equations:

  if $\sin\theta = q$, then $\theta = n\pi + (-1)^n \sin^{-1}q$
  if $\cos\theta = q$, then $\theta = 2n\pi \pm \cos^{-1}q$
  if $\tan\theta = q$, then $\theta = n\pi + \tan^{-1}q$

- Derivatives:

  $$\frac{d}{dx}\left[\sin^{-1}x\right] = \frac{1}{\sqrt{1-x^2}}$$
  $$\frac{d}{dx}\left[\cos^{-1}x\right] = \frac{-1}{\sqrt{1-x^2}}$$
  $$\frac{d}{dx}\left[\tan^{-1}x\right] = \frac{1}{1+x^2}$$

- Integrals:

  $$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) = -\cos^{-1}\left(\frac{x}{a}\right)$$
  $$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$