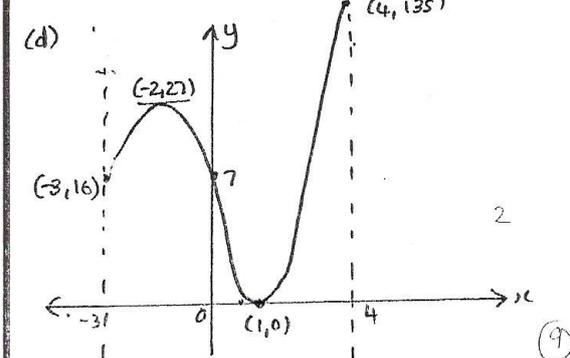


①  $y = x^3 + 5x^2 + 7x + 2$   
 $\frac{dy}{dx} = 3x^2 + 10x + 7$   
 when  $x=3$ ,  $\frac{dy}{dx} = 3(9) + 10(3) + 7 = 64 > 0$   
 $\therefore$  increasing when  $x=3$  (2)

②  $y = 2x^2 - 9x + 4$   
 $\frac{dy}{dx} = 4x - 9$   
 Decreasing when  $\frac{dy}{dx} < 0 \therefore 4x - 9 < 0$   
 $x < 9/4$  (2)

③  $y = 2x^3 + 3x^2 - 12x + 7$   
 $y' = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$   
 $= 6(x+2)(x-1)$   
 $y'' = 12x + 6 = 6(2x+1)$   
 (a) stationary when  $y' = 0 \therefore x = -2, 1$  (3)  
 (b) when  $x = -2$ ,  $y'' = (+)(-) < 0$   
 and  $y = 2(-8) + 3(4) - 12(-2) + 7 = 27$   
 $\therefore (-2, 27)$  is a max. t. pt.  
 and when  $x = 1$ ,  $y'' = (+)(+) > 0$   
 and  $y = 2 + 3 - 12 + 7 = 0$  (2)  
 $\therefore (1, 0)$  is a min. t. pt.

(c) when  $x = -3$ ,  $y = 2(-27) + 3(9) - 12(-3) + 7 = 16$   
 when  $x = 4$ ,  $y = 2(64) + 3(16) - 12(4) + 7 = 135$   
 $\therefore$  absolute minimum 0 when  $x = 1$  (2)  
 absolute maximum is 135 when  $x = 4$ .



④  $y = x^3 - 3x^2 + 3x + 1$   
 $y' = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1)$   
 $= 3(x-1)^2$   
 $y'' = 6x - 6 = 6(x-1)$   
 stationary when  $y' = 0$  i.e.  $x = 1$  (3)  
 when  $x = 1$ ,  $y = 1 - 3 + 3 + 1 = 2$  and  $y'' = 0$

$\therefore (1, 2)$  is a horizontal pt. of inflexion

⑤ (a)  $y = x^6 + 3x^5 - 4x^3 + 2x^2 - 9x - 7$   
 $y' = 6x^5 + 15x^4 - 12x^2 + 4x - 9$   
 $y'' = 30x^4 + 60x^3 - 24x + 4$  (2)

(b)  $y = (2x^2 + 5)^8$   
 $y' = 8(2x^2 + 5)^7 \cdot 4x = 32x(2x^2 + 5)^7$   
 $y'' = 32x \cdot 7(2x^2 + 5)^6 \cdot 4x + (2x^2 + 5)^7 \cdot 32$   
 $= 996x^2(2x^2 + 5)^6 + 32(2x^2 + 5)^7$   
 $= 32(2x^2 + 5)^6 [28x^2 + 2x^2 + 5]$  (2) (4)  
 $= 32(2x^2 + 5)^6 (30x^2 + 5)$   
 $= 160(2x^2 + 5)^6 (6x^2 + 1)$

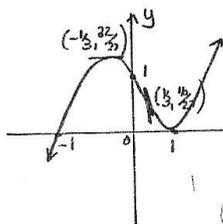
⑥  $y = 2x^2 - 2x - 1$   
 $y' = 4x - 2$   
 $y'' = 4$   
 $\therefore y + y' + y'' = 2x^2 - 2x - 1 + 4x - 2 + 4$   
 $= 2x^2 + 2x + 1$  (3)

⑦  $y = (x+1)(x-2)^2$   
 $y' = (x+1) \cdot 2(x-2) + (x-2)^2 \cdot 1$   
 $= (x-2)[2x+2+x-2]$   
 $= (x-2)(3x)$   
 $= 3x^2 - 6x$   
 $y'' = 6x - 6$   
 Concave down when  $y'' < 0$   
 $\therefore 6x - 6 < 0$   
 $6x < 6$   
 $x < 1$  (3)

⑧  $y = x^3 + x - 3$   
 $y' = 3x^2 + 1$   
 $y'' = 6x$   
 possible inflexion when  $y'' = 0$   
 $\therefore x = 0$  ( $y = -3$ )  
 for  $x < 0$ ,  $y'' = (+)(-) < 0$   
 for  $x > 0$ ,  $y'' = (+)(+) > 0$   
 $\therefore$  change of concavity  
 $\therefore (0, -3)$  is a pt. of inflexion (3)

⑨  $y = (x-1)(x^2-1)$   
 $y' = (x-1) \cdot 2x + (x^2-1) \cdot 1$   
 $= 2x^2 - 2x + x^2 - 1$   
 $= 3x^2 - 2x - 1 = (3x+1)(x-1)$   
 $y'' = 6x - 2 = 2(3x-1)$

stationary when  $y' = 0 \therefore x = 1, -\frac{1}{3}$   
 + when  $x = 1, y = 0, y'' = (+)(+) > 0$   
 $\therefore (1, 0)$  is a min. t. pt.  
 \* when  $x = -\frac{1}{3}, y = (-\frac{1}{3}-1)(\frac{1}{3}-1) = -\frac{4}{3} \times -\frac{2}{3} = \frac{8}{9}$   
 $y'' = (+)(-) < 0$   
 $\therefore (-\frac{1}{3}, \frac{8}{9})$  is a max. t. pt.  
 \* Inflection when  $y'' = 0$   
 $\therefore x = \frac{1}{3}$   
 when  $x < \frac{1}{3}$  (say  $x = 0.2$ )  
 $y'' = (+)(-) < 0$   
 when  $x > \frac{1}{3}$  (say  $x = 0.4$ )  
 $y'' = (+)(+) > 0$   
 when  $x = \frac{1}{3}, y = (\frac{1}{3}-1)(\frac{1}{3}-1) = \frac{4}{9}$   
 $\therefore (\frac{1}{3}, \frac{4}{9})$  inf. point

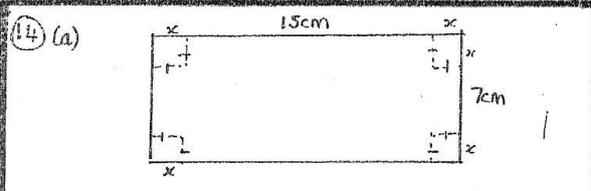


10)  $x + y = 25 \therefore y = 25 - x$   
 $P = xy = x(25 - x) = 25x - x^2$   
 $P' = 25 - 2x$   
 $P'' = -2$   
 when  $P' = 0, 25 - 2x = 0$   
 $x = 12.5$   
 at this time  $P'' < 0 \therefore$  Product is maximised  
 $\therefore P = 12.5(25 - 12.5) = 156.25$

11)  $y = (25 - x^2)^{1/2}$   
 $\frac{dy}{dx} = \frac{1}{2}(25 - x^2)^{-1/2} \cdot -2x = \frac{-x}{\sqrt{25 - x^2}}$   
 when  $x = 3, \frac{dy}{dx} = \frac{-3}{\sqrt{25 - 9}} = \frac{-3}{4}$   
 and  $y = \sqrt{25 - 9} = 4$   
 $\therefore y - 4 = \frac{-3}{4}(x - 3)$   
 $4y - 16 = -3x + 9$   
 $3x + 4y - 25 = 0$

12)  $y = x^3 - 5x^2 + 4x + 6$   
 $y' = 3x^2 - 10x + 4$   
 when  $x = 1, y' = 3 - 10 + 4 = -3$   
 $\therefore y - 6 = \frac{1}{3}(x - 1)$   
 $y - 6 = \frac{1}{3}x - \frac{1}{3}$   
 $3y - 18 = x - 1$   
 $x - 3y + 17 = 0$

13) (a)  $y = \frac{x^3}{3} + \frac{7x^2}{2} - 5x + c$   
 (b)  $y = x^4 - 3x^3 - 3x^2 + 6x + c$   
 (c)  $y = -\frac{1}{x} + c$



(b) length =  $15 - 2x$   
 breadth =  $7 - 2x$   
 height =  $x$   
 (c)  $V = lbh$   
 $= (15 - 2x)(7 - 2x)x$   
 $= (105 - 30x - 14x + 4x^2)x$   
 $= 105x - 44x^2 + 4x^3$   
 $\frac{dV}{dx} = 105 - 88x + 12x^2 = (2x - 3)(6x - 35)$   
 $\frac{d^2V}{dx^2} = -88 + 24x = 8(3x - 11)$   
 stationary when  $V' = 0$  ie  $x = \frac{3}{2}, \frac{35}{6}$   
 when  $x = \frac{3}{2}, V'' = (+)(-) < 0$   
 when  $x = \frac{35}{6}, V'' = (+)(+) > 0$   
 $\therefore$  volume maximum when  $x = \frac{35}{6}$

15)  $y = \frac{x^2}{x^2 - 4}$   
 $y' = \frac{(x^2 - 4) \cdot 2x - x^2(2x)}{(x^2 - 4)^2}$   
 $= \frac{2x^3 - 8x - 2x^3}{(x^2 - 4)^2} = \frac{-8x}{(x^2 - 4)^2}$   
 $y'' = \frac{(x^2 - 4)^2 \cdot -8 - (-8x) \cdot 2(x^2 - 4) \cdot 2x}{(x^2 - 4)^4}$   
 $= \frac{-8(x^2 - 4)^2 + 32x^2(x^2 - 4)}{(x^2 - 4)^4}$   
 $= \frac{-8(x^2 - 4)[(x^2 - 4) - 4x^2]}{(x^2 - 4)^4}$   
 $= \frac{-8(-4 - 3x^2)}{(x^2 - 4)^3}$   
 $= \frac{8(4 + 3x^2)}{(x^2 - 4)^3}$

(a) stationary when  $y' = 0 \therefore x = 0$   
 when  $x = 0, y = 0, y'' = \frac{(+)(+)}{(-)} < 0$   
 $\therefore (0, 0)$  is a max. t. pt.  
 (b) asymptotes where  $x^2 - 4 = 0$   
 $(x + 2)(x - 2) = 0$   
 $x = 2, -2$

