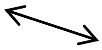
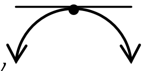
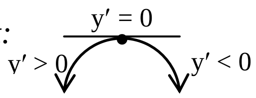
<u>Geometric Applications of the Derivative</u>

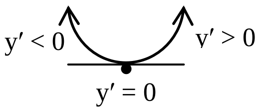
- **Types of Functions**
  - $\frac{dy}{dx} > 0$, increasing function 
  - $\frac{dy}{dx} < 0$, decreasing function 
  - $\frac{dy}{dx} = 0$, stationary point 
  - Monotonic increasing if $\frac{dy}{dx} > 0$ over particular interval
  - Monotonic decreasing if $\frac{dy}{dx} < 0$ over particular interval

- **Maximum and Minimum Points**
  - *Absolute maximum:* largest value of function in whole domain
  - *Relative maximum:* function is greater than other points in the neighbourhood
  - *Absolute minimum:* least value of function in whole domain
  - *Relative minimum:* function is smaller than other points in the neighbourhood
  - *Critical points* when f′ (x) = 0 and is undefined (denominator)

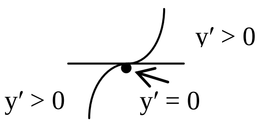
- **Stationary Points**
  - *Maximum turning point:*



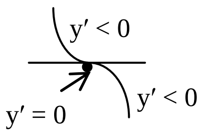
  - *Minimum turning point:*



  - *Horizontal point of inflexion on Rising Curve:*



  - *Horizontal point of inflexion on Falling Curve:*



- **Higher Derivatives**

  $y' = \frac{d}{dx}[f(x)]$

  $y'' = \frac{d}{dx}\left[\frac{dy}{dx}\right]$ *ie derivative of first derivative*

- **Second Derivative, Concavity, Turning Points and Points of Inflexion**
  - If y″ > 0, then the curve is concave upwards, and minimum turning point
  - If y″ < 0, then the curve is concave downwards and maximum turning point
  - If y″ = 0, and changes from y″ > 0 to y″ < 0 around, then it is a point of inflexion on rising curve
  - If y″ = 0, and changes from y″ < 0 to y″ > 0 around, then it is a point of inflexion on falling curve

- **Further Curve Sketching**
  - Find any stationary points using $f'(x) = 0$ and determine their nature
  - Find *possible* points of inflexion using $f''(x) = 0$ and determine their nature
  - Find intercepts on both axes:
    - for x-intercept $y = 0$
    - for y-intercept $x = 0$
  - Find the domain and range if applicable
  - Find any asymptotes or limits.
  - Use symmetry, particularly if the function is even or odd
  - Draw up a table of values as a last resort

- **Practical Applications**
  - Express the quantity to be maximised or minimised in terms of one variable
  - Differentiate with respect to the one variable
  - Make the derivate function equal to zero
  - Solve the equation
  - Test the change of sign of the first derivative of the sign of the second derivative test

- **Primative Function**
  - PF of x^n is $\frac{x^{n+1}}{n+1} + C$
  - If $f'(x) = (ax + b)^n$ then the PF is $\frac{(ax+b)^{n+1}}{a(n+1)} + C$