SOLUTIONS

1. (a) $y' = 27x^8 - 16x^7$ 1

(b) $y = x^{-7} \therefore y' = -7x^{-8} = \frac{-7}{x^8}$ 2

(c) $y = 4x^{\frac{1}{2}} \therefore y' = 4 \times \frac{1}{2} x^{-\frac{1}{2}} = 2x^{-\frac{1}{2}} = \frac{2}{\sqrt{x}}$ 2

(d) $y = 4x^{-\frac{1}{3}} \therefore y' = 4 \times -\frac{1}{3} x^{-\frac{4}{3}} = -\frac{4}{3} \times \frac{1}{x^{4/3}}$

$= \frac{-4}{3\sqrt[3]{x^4}}$ 2

(e) $y' = 5(4x+3)^4 . 4 = 20(4x+3)^4$ 1

(f) $y' = 7(x^3-5)^6 . 3x^2 = 21x^2(x^3-5)^6$ 1

(g) $y = (16-x^2)^{\frac{1}{2}} \therefore y' = \frac{1}{2}(16-x^2)^{-\frac{1}{2}} . -2x$

$= -x(16-x^2)^{-\frac{1}{2}}$

$= \frac{-x}{\sqrt{16-x^2}}$ 2

(h) $y = (2x+3)^{-6} \therefore y' = -6(2x+3)^{-7} . 2$

$= \frac{-12}{(2x+3)^7}$ 2

(i) $y' = (x-2).1 + (x+7).1$

$= x - 2 + x + 7$

$= 2x + 5$ 2

(j) $y' = x . 5(x^2-1)^4 . 2x + (x^2-1)^5 . 1$

$= 10x^2(x^2-1)^4 + (x^2-1)^5$

$= (x^2-1)^4 [10x^2 + x^2 - 1]$

$= (x^2-1)^4 (11x^2-1)$ 3

(k) $y' = (x-1)^2 . 3(x+2)^2 + (x+2)^3 . 2(x-1)$

$= (x-1)(x+2)^2 [3(x-1) + 2(x+2)]$

$= (x-1)(x+2)^2 (3x-3+2x+4)$ 3

$= (x-1)(x+2)^2 (5x+1)$

(l) $y' = \frac{(3x-4).2 - (2x+9).3}{(3x-4)^2}$

$= \frac{6x-8-6x-27}{(3x-4)^2}$ 3

$= \frac{-35}{(3x-4)^2}$ (24)

2. (a) $P'(x) = 8x \therefore P'(2) = 8 \times 2 = 16$ 2 (4)

(b) $G'(x) = -3x^2 \therefore G'(-1) = -3 \times (-1)^2 = -3$ 2

3. $\frac{dp}{dq} = 8q$ (1)

4. $y = x^2 + 3x \therefore y' = 2x + 3$ 1

when $x = 3$, $y = 3^2 + 3 \times 3 = 18$

$y' = 2 \times 3 + 3 = 9$ (grad. of tangent)

$\therefore y - 18 = 9(x-3)$

$y - 18 = 9x - 27$

$y = 9x - 9$ (4)

5. $f(x) = 3 - x^2 \therefore f'(x) = -2x$ 1

when $x = -1$, $f'(x) = -2x - 1 = 2$ 1

$\therefore$ grad. of tangent is 2

and grad. of normal is $-\frac{1}{2}$

$\therefore$ normal is $y - 2 = -\frac{1}{2}(x+1)$

$2y - 4 = -(x+1)$

$2y - 4 = -x - 1$ (4)

$x + 2y - 3 = 0$

6. (a) $f'(x) = 2x - 6$ 1

(b) $f'(x) = 0 \therefore 2x - 6 = 0$ 1

$2x = 6$

$x = 3$ (4)

when $x = 3$, $y = 3^2 - 6 \times 3 + 5 = -4$

$\therefore$ point is $(3, -4)$ 1

7. (a) $y = x^2 - 9$ ... (1)

$y = x^2 - 8x + 15$ .. (2)

equate (1) and (2) $x^2 - 9 = x^2 - 8x + 15$

$8x = 24$

$x = 3$

when $x = 3$, $y = 3^2 - 9 = 0$ 2

$\therefore$ point is $(3, 0)$

(b) $\frac{dy}{dx} = 2x = 2 \times 3 = 6 \therefore m_1 = 6$

$\frac{dy}{dx} = 2x - 8 = 2 \times 3 - 8 = -2 \therefore m_2 = -2$ 2

P.T.O.

(c) from (b) $m_1 = 6$ and $m_2 = -2$

$$\therefore \tan\theta = \left|\frac{6 - -2}{1 + 6 \times -2}\right|$$

$$= \left|\frac{8}{-11}\right|$$

$$= \frac{8}{11}$$

$\therefore \theta = 36°2'$

(8) $\frac{dy}{dx} = 2x - 5$

but $\frac{dy}{dx} = -3$ $\quad \therefore 2x - 5 = -3$

$2x = 2$

$x = 1$

when $x = 1$, $y = 1^2 - 5 \times 1 = 1 - 5 = -4$.

$\therefore$ point of contact is $(1, -4)$