Year 12 Extension 1 **Inverse Functions Assignment** **Date Due:**

1. Find the inverse function for the following functions and state the domain and range for the inverse in each case:

   (a) $y = 3 - 2x$ (b) $y = \frac{1}{x}$ (c) $y = x^2 - 2$ (d) $y = \frac{1}{x+2}$

2. For the function $y = 3x^2$,
   (a) state the restricted domain so that it will have an inverse function.
   (b) find the inverse function.
   (c) state the domain of the inverse function.

3. If $xy^2 = 9$, by expressing y as a function of x, find:
   (a) $\frac{dy}{dx}$ in terms of x and hence
   (b) $\frac{dx}{dy}$ in terms of x.
   (c) By expressing x as a function of y find $\frac{dx}{dy}$ directly and so check the answer to part (b).

4. Evaluate the following and given your answers in terms of π

   (a) $\sin^{-1}1$ (b) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}\left(-\sqrt{3}\right)$

   (e) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (f) $\tan^{-1}0$ (g) $\sin^{-1}\left(\frac{-1}{2}\right)$ (h) $\cos^{-1}\left(\frac{-1}{2}\right)$

5. Without the use of a calculator, evaluate the following: (a) $\sin^{-1}\left(\sin\frac{\pi}{3}\right)$ (b) $\sin\left[\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)\right]$

6. Find the value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$.

7. For each of the following state the function and range: (a) $y = \sin^{-1}2x$ (b) $\tan^{-1}\left(\frac{x}{2}\right)$.

8. Evaluate $\cos\left[\sin^{-1}\left(\frac{9}{41}\right)\right]$ and give tour answer in exact form.

9. Differentiate: (a) $3\sin^{-1}$ (b) $\tan^{-1}(5x)$ (c) $\cos^{-1}(x^2)$ (d) $\tan^{-1}\left(\frac{x}{4}\right)$

10. Find the equations of the tangent and the normal to $y = \sin^{-1}(x - 1)$ at the point $\left(\frac{3}{2}, \frac{\pi}{6}\right)$.

11. Find the following indefinite integrals: (a) $\int \frac{dx}{x^2+9}$ (b) $\int \frac{dx}{\sqrt{1-2x^2}}$.

12. Evaluate the following: (a) $\int_{-1}^{1} \frac{dx}{\sqrt{2-x^2}}$ (b) $\int_0^1 \frac{dx}{x^2+1}$ (c) $\int_0^3 (9-x^2)^{\frac{-1}{2}}\,dx$

13. Find the area under the curve $y = \frac{16}{x^2+4}$ between the lines $x = -2$ and $x = 2$.

14. The area bounded by the curve $y = (1 + 4x^2)^{-½}$, the x-axis and the ordinates $x = 0$ and $x = \frac{\sqrt{3}}{2}$ is rotated about the x-axis. Find the volume generated.

15. Use the Trapezoidal Rule with 4 sub-intervals to find an approximation to the area under the curve $y = \sin^{-1}x$ and between $x = 0$ and $x = 1$.