Year 12 Extension 1 **Inverse Trigonometric Functions Assignment** **Date Due:**

1. Find functions which are the inverse of each of the following. State that domain and range of the inverse in each case.

   (a) $f(x) = 2x - 4$ (b) $f(x) = x^2 - 1, x \geq 0$ (c) $g(x) = \sqrt{x-3}$

2. Show that $f(x) = 2x - 1$ and $g(x) = \frac{1}{2}(x + 1)$ are inverses by showing that $f[g(x)] = g[f(x)] = x$.

3. Find the exact value of:

   (a) $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ (b) $\sin^{-1}\left(-\frac{1}{2}\right)$ (c) $\sin^{-1}(\sin 1.2)$ (d) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

   (e) $\sin\left[\cos^{-1}\left(\frac{1}{2}\right)\right]$ (f) $\tan^{-1}(-\sqrt{3})$ (g) $\tan\left[\tan^{-1}\left(\frac{-5}{13}\right)\right]$ (h) $\cos\left[\sin^{-1}\left(\frac{1}{3}\right)\right]$

4. Evaluate $\cos\left[\sin^{-1}\frac{5}{13} + \sin^{-1}\frac{4}{5}\right]$

5. Show that $\tan^{-1}4 - \tan^{-1}\frac{3}{5} = \frac{\pi}{4}$

6. Sketch the graphs of the following, stating their largest possible domain and range:

   (a) $y = 2\tan^{-1}x$ (b) $y = 2\cos^{-1}x - 1$ (c) $y = \sin^{-1}\frac{x}{2}$

7. Differentiate: (a) $f(x) = \sin^{-1}\frac{1}{2}x$ (b) $f(x) = \cos^{-1}(1 - 2x)$

   (c) $f(x) = \tan^{-1}(3x + 1)$ (d) $f(x) = \sin^{-1} \cdot \cos^{-1}\frac{x}{2}$

8. (a) Find: (i) $\int \frac{dx}{\sqrt{9 + 16x^2}}$ (ii) $\int \frac{dx}{x^2 + 16}$

   (b) Evaluate: (i) $\int_0^1 \frac{dx}{\sqrt{2 - x^2}}$ (ii) $\int_0^{\sqrt{3}} \frac{dx}{\sqrt{3 - x^2}}$

9. Find the area bounded by the curve $y = \frac{1}{x^2 + 1}$, the x-axis and the ordinates $x = 1$ and $x = -1$.

10. (a) Find the volume of the solid of revolution formed by rotating the curve $y = \sin x$ between $x = 0$ and $x = \frac{\pi}{4}$ about the x-axis.

    (b) Hence, find the volume of the solid formed by rotating the curve $y = \sin^{-1}x$ between $y = 0$ and $y = \frac{\pi}{2}$ about the y-axis.