MATHEMATICS REVISION OF FORMULAE AND RESULTS

Surds

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$
- $(\sqrt{a})^2 = a$

Absolute Value

$$|a| = a \text{ if } a \geq 0$$
$$|a| = -a \text{ if } a < 0$$

Geometrically:

$|x|$ is the distance of *x* from the origin on the number line
$|x - y|$ is the distance between *x* and *y* on the number line

$$|ab| = |a|.|b|$$

$$|a + b| \leq |a| + |b|$$

Factorisation

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

Real Functions

- A function is ***even*** if $f(-x) = f(x)$. The graph is symmetrical about the y-axis.
- A function is ***odd*** if $f(-x) = -f(x)$. The graph has point symmetry about the origin.

The Circle

The equation of a circle with:

- Centre the origin (0, 0) and radius *r* units is:

$$x^2 + y^2 = r^2$$

- Centre (*a*, *b*) and radius *r* units is:

$$(x - a)^2 + (y - b)^2 = r^2$$

Co-ordinate Geometry

- Distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Gradient formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \tan\theta$
- Midpoint Formula: midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- Perpendicular distance from a point to a line:

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

- Acute angle between two lines (or tangents)

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

- Equations of a Line

  gradient-intercept form: $y = mx + b$

  point-gradient form: $y - y_1 = m(x - x_1)$

  two point formula: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

  intercept formula: $\frac{x}{a} + \frac{y}{b} = 1$

  general equation: $ax + by + c = 0$

- Parallel lines: $m_1 = m_2$
- Perpendicular lines: $m_1 . m_2 = -1$

Trigonometric Results

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ (SOH)
- $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ (CAH)
- $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ (TOA)
- Complementary ratios:

$$\sin(90° - \theta) = \cos\theta$$

$$\cos(90° - \theta) = \sin\theta$$

$$\tan(90° - \theta) = \cot\theta$$

$$\sec(90° - \theta) = \operatorname{cosec}\theta$$

$$\operatorname{cosec}(90° - \theta) = \sec\theta$$

- Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

- The Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\text{Cos}A$$

$$\text{Cos}A = \frac{b^2 + c^2 - a^2}{2bc}$$

- The Area of a Triangle

$$\text{Area} = \frac{1}{2}ab\text{Sin}C$$

The Quadratic Polynomial

- The general quadratics is: $y = ax^2 + bx + c$
- The quadratic formula is: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- The discriminant is: $\Delta = b^2 - 4ac$

  If $\Delta \geq 0$ the roots are real

  If $\Delta < 0$ the roots are not real

  If $\Delta = 0$ the roots are equal

  If Δ is a perfect square, the roots are rational

- If α and β are the roots of the quadratic equation

$$ax^2 + bx + c = 0$$

  then: $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

- The axis of symmetry is: $x = -\frac{b}{2a}$
- If a quadratic function is positive for all values of x, it is *positive definite* i.e. $\Delta < 0$ and $a > 0$
- If a quadratic function is negative for all values of x, it is *negative definite* i.e. $\Delta < 0$ and $a < 0$
- If a function is sometimes positive and sometimes negative, it is *indefinite* i.e. $\Delta > 0$

The Parabola

- The parabola $x^2 = 4ay$ has vertex $(0,0)$, focus $(0,a)$, focal length ‘a’ units and directrix $y = -a$
- The parabola $(x - h)^2 = 4a(y - k)$ has vertex (h, k)

Differentiation

- First Principles:

$$f'(x) = \lim_{h \to \infty} \frac{f(x+h) - f(x)}{h}$$

or

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{h}$$

- If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$
- Chain Rule: $\frac{d}{dx}[f(u)] = f'(u)\frac{du}{dx}$
- Product Rule: If $y = uv$ then $\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
- Quotient Rule: If $y = \frac{u}{v}$ then $\frac{dy}{dx} = \frac{v\frac{du}{dx} + u\frac{dv}{dx}}{v^2}$
- Trigonometric Functions:

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

- Exponential Functions: $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)e^{f(x)}$

$$\frac{d}{dx}(a^x) = a^x.\ln a$$

- Logarithmic Functions: $\frac{d}{dx}\left(\log_e f(x)\right) = \frac{f'(x)}{f(x)}$

Geometrical Applications of Differentiation

- Stationary points: $\frac{dy}{dx} = 0$
- Increasing function: $\frac{dy}{dx} > 0$
- Decreasing function: $\frac{dy}{dx} < 0$
- Concave up: $\frac{d^2y}{dx^2} < 0$
- Concave down: $\frac{d^2y}{dx^2} > 0$
- Minimum turning point: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} > 0$
- Maximum turning point: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} < 0$
- Points of inflexion: $\frac{d^2y}{dx^2} = 0$ and concavity changes about the point.
- Horizontal points of inflexion: $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 0$ and concavity changes about the point.

Approximation Methods

- The Trapezoidal Rule:

$$\int_a^b f(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + y_3 + \ldots + y_{n-1})]$$

- Simpson's Rule:

$$\int_a^b f(x)dx = \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)]$$

In both rules, $h = \frac{b-a}{n}$ where n is the number of strips.

Integration

- If $f(x) \geq 0$ for $a \leq x \leq b$, the area bounded by the curve $y = f(x)$, the x-axis and $x = a$ and $x = b$ is given by $\int_a^b f(x)\,dx$.
- The volume obtained by rotating the curve $y = f(x)$ about the x-axis between $x = a$ and $x = b$ is given by $\pi \int_a^b [f(x)]^2$
- If $\frac{dx}{dx} = x^n$ then $y = \frac{x^{n+1}}{n+1}$
- If $\frac{dx}{dx} = (ax+b)^n$ then $y = \frac{(ax+b)^n}{a(n+1)}$
- Trigonometric Functions:

$$\int \sin x\,dx = -\cos x + \mathrm{C}$$

$$\int \cos x\,dx = \sin x + \mathrm{C}$$

$$\int \sec^2 x\,dx = \tan x + \mathrm{C}$$

- Exponential Functions:

$$\int e^{ax}\,dx = \frac{e^{ax}}{a} + \mathrm{C} \quad \text{and} \quad \int a^x\,dx = \frac{1}{\ln a}.a^x$$

- Logarithmic Functions:

$$\int \frac{f'(x)}{f(x)}dx = \log_e x + \mathrm{C}$$

Sequences and Series

- Arithmetic Progression

$$d = U_2 - U_1$$

$$U_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$S_n = \frac{n}{2}[a + l]$ where l is the last term

- Geometric Progression

$$r = \frac{U_2}{U_1}$$

$$U_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r}$$

The Trigonometric Functions

- π radians = 180°
- Length of an arc: $l = r\theta$
- Area of a sector: $A = \frac{1}{2}r^2\theta$
- Area of a segment: $A = \frac{1}{2}r^2(\theta - \sin\theta)$

  [In these formulae, θ is measured in radians.]
- Small angle results:

$$\sin x \to 0$$
$$\cos x \to 1$$
$$\tan x \to 0$$
$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\tan x}{x} = 1$$

- For $y = \sin nx$ and $y = \cos nx$ the period is $\frac{2\pi}{n}$
- For $y = \sin nx$ the period is $\frac{\pi}{n}$

Logarithmic and Exponential Functions

- The Index Laws:

$a^x \times a^y = a^{x+y}$

$a^x \div a^y = a^{x-y}$

$(a^x)^y = a^{xy}$

$a^{-x} = \frac{1}{a^x}$

$a^{\frac{x}{y}} = \sqrt[y]{a^x}$

$a^0 = 1$

- The logarithmic Laws:

If $\log_a b = c$ then $a^c = b$

$\log_a x + \log_a y = \log_a xy$

$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

$\log_a x^n + n\log_a x$

$\log_a a = 1$ and $\log_a 1 = 0$

- The Change of Base Result:

$\log_a b = \frac{\log_e b}{\log_e a} = \frac{\log_{10} b}{\log_{10} a}$