DAPTO HIGH SCHOOL

STRIVE FOR HIGHER THINGS

2009
HSC Preliminary Course
FINAL EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using a blue or black pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown for every question
- Begin each question on a fresh sheet of paper

Total marks (80)

- Attempt Questions 1 – 7
- Q 1 – 5 are 12 marks each
- Q6 and 7 are 10 marks each

Question 1 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) Find the value of $\frac{1}{7.38} + \frac{1}{9.85}$, correct to 3 significant figures. **2**

(b) Express the decimal $0.\dot{4}\dot{8}$ as a fraction in simplest form. **2**

(c) If $\sqrt{56} + \sqrt{14} = \sqrt{A}$, find A. **2**

(d) Find the exact value of $t^4 - 2t^2 + 1$ when $t = 2\sqrt{5}$. **2**

(e) Factorise the following expressions fully:

(i) $x^2 - 5x - 14$ **1**

(ii) $(x - 1)^2 - 16$ **1**

(f) Simplify $\frac{10x - 15}{6} \times \frac{1}{8x - 12}$ as a single fraction in simplest form. **2**

End of Question 1

Question 2 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) $f(x) = 2x^2 - x$:

(i) Find $f(-x)$ **1**

(ii) Is this an even function, odd function or neither? **1**

(b) A function is defined by the rule $g(x) = \begin{cases} x + 1 & \text{if } x \geq 1 \\ -1, & \text{if } -2 < x < 1 \\ 1 - x & \text{if } x \leq -2 \end{cases}$

Find,

(i) $g(1)$ **1**

(ii) $g(-1)$ **1**

(iii) $g(0)$ **1**

(iv) $g(2) + g(-2)$ **1**

(c) Sketch the graphs of the following, stating the domain and range of each.

(i) $y = 2^x$ **2**

(ii) $x^2 + (y + 3)^2 = 36$ **2**

(iii) $0 = 3x - y - 5$ **2**

End of Question 2

Question 3 **(12 Marks)** Use a Separate Sheet of paper **Marks**

(a) From a point 5m above the ground, the angle of depression of the bottom of a wall is 21° and the angle of elevation of the top of the wall is 32°.

(i) Draw a diagram to show this information. **1**

(ii) Find the distance from the point of observation to the bottom of the wall.(correct to 2 decimal places) **2**

(iii) Using your answer from part (ii) and the Sine Rule, find the height of the wall. (correct to 2 decimal places) **2**

(b) Zoe and Kobi set out on a bike ride from point P at the same time. One travels at 20km/h along a straight road in the direction 032°T. The other travels at 25km/h along another straight road in the direction 132°T.

(i) Draw a diagram to represent this information. **1**

(ii) Find the distance Zoe and Kobi are apart to the nearest kilometre after 3 hours. **2**

(c) (i) Solve $4\sin^2\theta = 3$ for $-180° \le \theta \le 180°$. **2**

(ii) Prove $\sec\theta + \tan\theta = \dfrac{1 + \sin\theta}{\cos\theta}$. **2**

End of Question 3

Question 4 **(12 Marks)** Use a Separate Sheet of paper **Marks**

The points A(2, 0), B(8, 4), C(4, 6) and D(x_1, y_1) form the 4 vertices of a parallelogram.

(a) Draw a number plant and mark *A*, *B*, & *C* on it. **1**

(b) Find the gradient of *AB*. **1**

(c) Show that the equation of the line *l* parallel to *AB* and going through *C* is $2x - 3y + 10 = 0$. **2**

(d) If the equation of the line *k* through *A* parallel to *BC* is $x + 2y - 2 = 0$. Find the point $D(x_1, y_1)$ the intersection of the lines *l* and *k*. Mark this point on your diagram. **2**

(e) Find the angle θ to the nearest degree that the line *AB* makes with the positive *x*-axis. **2**

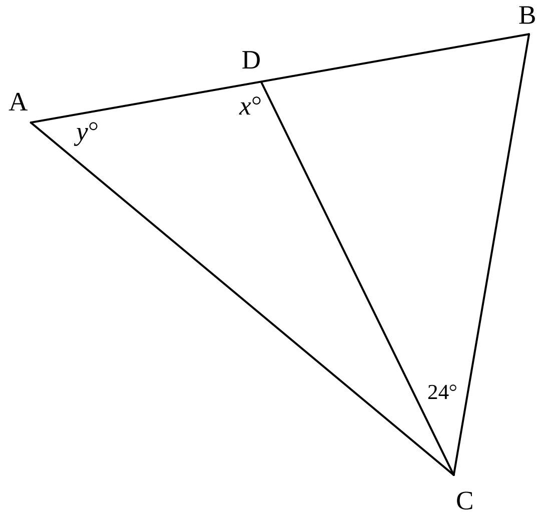
(f) Find the perpendicular distance between the line *l* and *A*. **2**

(g) Find the exact area of *ABCD*. **2**

End of Question 4

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

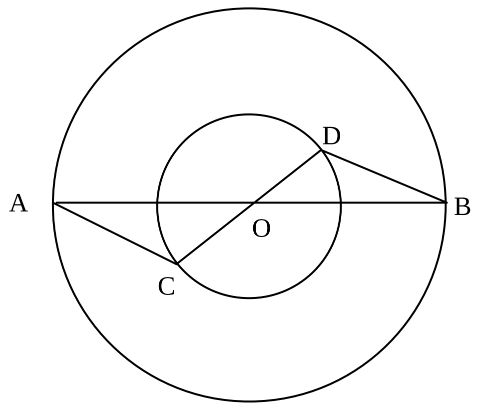
(a) In the figure below AB = BC = DC and $\angle BCD = 24°$.



NOT TO SCALE

Find the values of x and y, giving reasons for each step. **2**

(b) In the diagram below O is the centre of the two circles. AB is the diameter of the larger circle and CD is the diameter of the smaller circle.



(i) Prove that ΔAOC is congruent to ΔBOC. **3**

(ii) Hence, show that AC || DB. **1**

Question 5 continues on page 7

Question 5 **(12 Marks)** Use a Separate Sheet of paper **Marks**

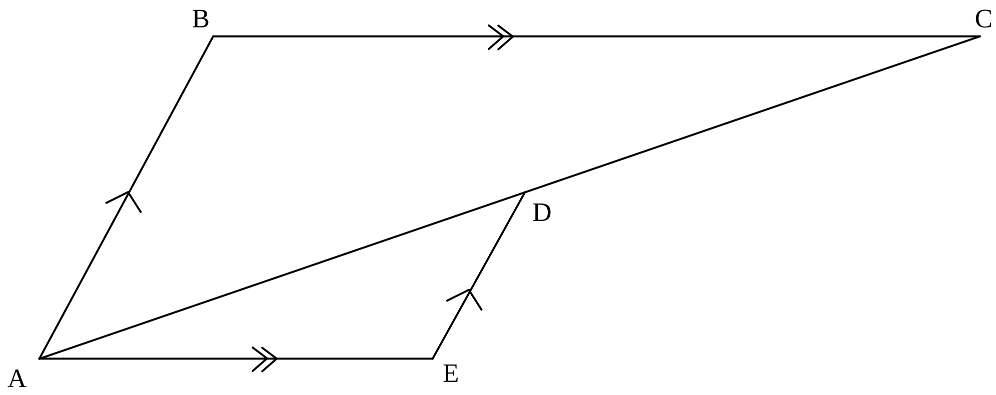
(c) The sum of the interior angles of a regular polygon is 2700°.

(i) How many sides has the polygon? **1**

(ii) Find the size of each interior angle to the nearest minute. **1**

(iii) Hence find the size of each interior angle. **1**

(d) Prove that ΔABC is similar to ΔDEA. **3**



End of Question 5

Question 6 **(10 Marks)** Use a Separate Sheet of paper **Marks**

(a) Solve:

(i) $\frac{a+2}{3} = \frac{a}{2} - 2$ **2**

(ii) $6(y - 1) = 3(y + 8)$ **2**

(b) Show the region of the number plane where the following hold simultaneously:

$y \le x + 1$

and $y > \frac{4}{x}$ **3**

(c) Find the exact value of sec(60°). **1**

(d) Solve: $4 + 3x - x^2 > 0$. **2**

End of Question 6

Question 7 **(10 Marks)** Use a Separate Sheet of paper **Marks**

(a) Solve:

(i) $|5x - 3| = |3x + 1|$ **2**

(ii) $2x^2 - 3x + 1 = 0$ **2**

(b) A rhombus has diagonals 6 cm and 8 cm.

(i) Find the area of the rhombus. **1**

(ii) Find the length of each side of the rhombus. **2**

(iii) Find the perimeter of the rhombus. **1**

(c) Find $\lim_{x \to 2} \frac{x - 3x}{x^2 - 4}$ **2**

End of Examination

STANDARD INTEGREALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 1$$

$$\int \frac{1}{x} dx = \text{In } x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a \neq 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \text{In}\left(x + \sqrt{x^2 - a^2}\right) \quad x < a < 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \text{In}\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\text{In}x = \log_e x, \quad x > 0$