ANSWERS

① (a)

$$
\begin{array}{r}
2x^2 + 3x - 5 \\
2x-3\;\big)\; 4x^3 \qquad -19x + 9 \\
4x^3 - 6x^2 \\
\hline
6x^2 - 19x \\
6x^2 - 9x \\
\hline
-10x + 9 \\
-5x + 15 \\
\hline
-6
\end{array}
$$

2

(b)

$$
\begin{array}{r}
x^2 + x - 6 \\
x^2-1\;\big)\; x^4 + x^3 - 7x^2 - x + 6 \\
x^4 \qquad - x^2 \\
\hline
x^3 - 6x^2 - x \\
x^3 \qquad - x \\
\hline
-6x^2 + 6 \\
-6x^2 + 6 \\
\hline
0
\end{array}
$$

2 4

② (a) $P(-2) = -8 + 3(4) + 2(-2) - 7$
$= -8 + 12 - 4 - 7$
$= -7$ 2

(b) $P(-\tfrac{3}{2}) = \frac{81}{16} + 2\left(\frac{9}{4}\right) - 13\left(\frac{-3}{2}\right) - 60$
$= \frac{81}{16} + \frac{18}{4} + \frac{39}{2} - 60$ 2 4
$= -30\tfrac{15}{16}$

③ $P(-2) = 9 \therefore -8 + 12 + 2m + n = 9$ 1
$2m + n = 5 \ldots$ ①

$P(3) = 49 \therefore 27 + 27 - 3m + n = 49$ 1
$-3m + n = -5 \ldots$ ②

① − ② $5m = 10$
$m = 2$

subs in ① $4 + n = 5 \therefore n = 1$ 1 3

Hence, $m = 2$, $n = 1$

④ $P(1) = 0 \therefore 1 + 4 + a - b = 0$
$a - b = -5 \ldots$ ① 1

$P(-2) = 0 \therefore 16 - 32 + 4a - b = 0$
$4a - b = 16 \ldots$ ② 1

② − ① $3a = 21$
$a = 7$

subs in ① $7 - b = -5$
$\therefore b = 12$ 1 3

Hence, $a = 7$ and $b = 12$

⑤ $P(-4) = 6(-64) + 35(16) + 34(-4) - 40 = 0$ 1

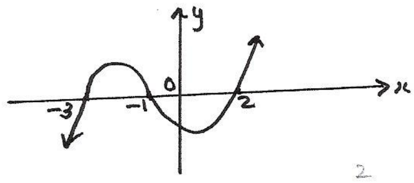
$\therefore (x+4)$ is a factor

$$
\begin{array}{r}
6x^2 + 11x - 10 \\
x+4\;\big)\; 6x^3 + 35x^2 + 34x - 40 \\
6x^3 + 24x^2 \\
\hline
11x^2 + 34x \\
11x^2 + 44x \\
\hline
-10x - 40 \\
-10x - 40 \\
\hline
0
\end{array}
$$

2

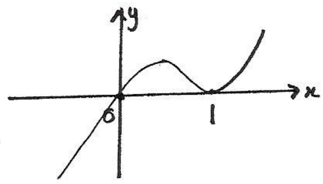
$\therefore P(x) = (x+4)(6x^2 + 11x - 10)$ 4
$= (x+4)(2x+5)(3x-2)$. 1

⑥ (a)



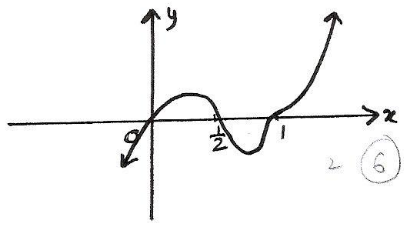
2

(b)



2

(c)



2 6

⑦ $x^3 + 2x^2 + 3x + 4 = 0$

$\gamma + \alpha + \beta = -2$, $\alpha\beta + \alpha\gamma + \beta\gamma = 3$, $\alpha\beta\gamma = -4$

(a) $(\alpha-1)(\beta-1)(\gamma-1) = (\alpha-1)(\beta\gamma - \beta - \gamma + 1)$
$= \alpha\beta\gamma - \alpha\beta - \alpha\gamma + \alpha - \beta\gamma + \beta + \gamma - 1$
$= \alpha\beta\gamma - (\alpha\beta + \alpha\gamma + \beta\gamma) + (\alpha + \beta + \gamma) - 1$
$= -4 - 3 - 2 - 1$
$= -10$ 2

(b) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$
$= (-2)^2 - 2(3)$
$= 4 - 6$ 2
$= -2$

(c) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma} = \frac{\beta\gamma+\alpha\gamma+\alpha\beta}{\alpha\beta\gamma}$

$= 3 \div -4$

$= -\frac{3}{4}$

(d) $\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma} = \frac{\gamma+\beta+\alpha}{\alpha\beta\gamma} = \frac{-2}{-4} = \frac{1}{2}$

(8) Let the roots be α, α and β

$\alpha+\alpha+\beta = 7/2 \quad \therefore \quad 2\alpha+\beta = 7/2 \; \ldots \; (1)$

$\alpha\beta+\alpha\alpha+\alpha\beta = \frac{12}{2} \quad \therefore \quad 2\alpha\beta+\alpha^2 = {}^-6 \; \ldots \; (2)$

$\alpha.\alpha.\beta = \frac{-45}{2} \quad \therefore \quad \alpha^2\beta = \frac{-45}{2} \; \ldots \; (3)$

from (1) $\beta = \frac{7}{2} - 2\alpha \; \ldots \; (4)$

subs (4) into (2) $2\alpha\left(\frac{7}{2}-2\alpha\right)+\alpha^2 = {}^-6$

$7\alpha - 4\alpha^2 + \alpha^2 = {}^-6$

$3\alpha^2 - 7\alpha - 6 = 0$

$(3\alpha+2)(\alpha-3) = 0$

$\therefore \alpha = 3, {}^-2/3$

when $\alpha = 3$, $\beta = \frac{7}{2} - 2(3) = \frac{-5}{2}$

when $\alpha = -\frac{2}{3}$, $\beta = \frac{7}{2} - 2\left(\frac{-2}{3}\right) = \frac{29}{6}$

(9) $x^3+qx+r=0$

$\Sigma\alpha = 0$, $\Sigma\alpha\beta = q$, $\Sigma\alpha\beta\gamma = r$

Let the roots be α, α and β

$\therefore 2\alpha+\beta = 0 \; \ldots \; (1)$

$2\alpha\beta+\alpha^2 = q \; \ldots \; (2)$

$\alpha^2\beta = -r \; \ldots \; (3)$

From (1) $\beta = -2\alpha \; \ldots \; (4)$

subs in (2) $2\alpha(-2\alpha)+\alpha^2 = q$

$-4\alpha^2+\alpha^2 = q$

$q = -3\alpha^2$

subs (4) into (3) $\alpha^2(-2\alpha) = {}^-r$

$+2\alpha^3 = r$

Now: $4q^3+27r^2 = 4\left(-3\alpha^2\right)^3 + 27\left(+2\alpha^3\right)^2$

$= 4\left(-27\alpha^6\right) + 27(4\alpha)^6$

$= -108\alpha^6 + 108\alpha^6$

$= 0$

$\therefore 4q^3+27r^2 = 0$

(10) $x^3 - 3x^2 - 4x + 12 = 0$

$\therefore \Sigma\alpha = 3$, $\Sigma\alpha\beta = -4$, $\Sigma\alpha\beta\gamma = -12$

Let the roots be $\alpha, -\alpha$ and β

$\therefore \alpha + -\alpha + \beta = 3 \quad \therefore \beta = 3$

$\alpha(-\alpha)+\alpha\beta+(-\alpha\beta) = -4 \quad \therefore -\alpha^2 = {}^-4$

$\alpha^2 = 4, \alpha = \pm 2$

$\alpha(-\alpha)(\beta) = -12 \quad \therefore -\alpha^2\beta = -12$

$\therefore$ the roots are $x = 2, -2, 3$