POLYNOMIALS TEST

1. If $P(x) = x^4 - 4x^2 + 3$, find the zeros of $P(x)$ over the field of:
   (a) rationals
   (b) reals
   (c) complex numbers

2. Express $P(x) = x^4 + 4x^3 - 3x^2 + 8x - 10$ as a product of irreducible factors over the complex field.

3. If $P(x) = 4x^3 + 15x^2 + 12x - 4$ has a double root, find all the roots of $P(x)$ over the real field.

4. Find the remainder when $P(x) = x^3 + 2x^2 - 1$ is divided by:
   (a) $x - i$
   (b) $x^2 + 1$

5. When $P(x) = x^4 + ax^2 + bx$ is divided by $x^2 + 1$, the remainder is $x + 2$. Find the values of a and b.

6. $P(x)$ is an even monic polynomial of degree 4 with integer coefficients. If $\sqrt{2}$ is a zero of $P(x)$ and the constant term is 6, find $P(x)$ in factored form.

7. The equation $x^3 + x^2 - 2x - 3 = 0$ has roots α, β, γ. Find the equation with roots:
   (a) $2\alpha, 2\beta, 2\gamma$
   (b) $\frac{\alpha}{2}, \frac{\beta}{2}, \frac{\gamma}{2}$
   (c) $\alpha - 2, \beta - 2, \gamma - 2$

8. The equation $x^3 + 2x + 1 = 0$ has roots α, β, γ. Evaluate:
   (a) $\alpha + \beta + \gamma$
   (b) $\alpha^2 + \beta^2 + \gamma^2$
   (c) $\alpha^3 + \beta^3 + \gamma^3$
   (d) $\alpha^4 + \beta^4 + \gamma^4$