Probability

- **Definition of Probability**
  $P(E) = \frac{n(E)}{n(S)}$

- **Events Which are likely to occur or are impossible**
  $0 \leq P(E) \leq 1$

- **Complementary Events**
  P(E) is the probability that an event occurs, $P(\tilde{E})$ is the probability that it does not occur.
  $P(E) + P(\tilde{E}) = 1$, $P(\tilde{E}) = 1 - P(E)$

- **Non mutually exclusive events**
  The occurrence of one event prevents the occurrence of another = mutually exclusive
  Occurrence does not prevent this = non mutually exclusive
  P(A or B) = P(A) + P(B) – P(A and B)

- **The product rule**
  When more than one event occurs, multiply the probabilities together.

- **Tree diagrams**
  P(A or B) = P(A U B) = P(A) + P(B)
  This is the addition rule of probability

- **Counting techniques**
  If one event can occur in *p* different ways and after this event can happen in *q* different ways than the successive outcomes can happen in *pq* different ways.

- **Ordered Selections of *n* Different Elements from a Set of *n* Such Elements**
  We use n! (n factorial) for such arrangements.
  The number of *n* different elements from a set of *n* suc elements is:
  n! Where $n! = n(n-1)(n-2)(n-3) \ldots 3 \times 2 \times 1$

- **Ordered Selections of *r* Different Elements from a Set of *n* Such Elements**
  If we select r different elements from a set of n such elements there would be:
  $\frac{n!}{(n-r)!}$ Ordered selections.
  This is donated by ${}^{n}P_{r}$ (permutation)
  The number of ordered sets (each with n elemts) from a set of n distinct elemts is donated by:
  ${}^{n}P_{n}$, being $\frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$

- **Ordered selections of n Elements not all of which are alike**
  The number of ordered selections of n elements, of which x are alike of one kind, y of another and z of another is:
  $\frac{n!}{z!\,y!\,z!}$

- **Arrangements about a circle**

If n different things are arranged in a circle, the number of permutations is $^1/_n$ of n! = (n – 1)!

- **Unordered selections**
  The number of unordered selections of r unlike elements from n unlike elements is given by ^nC_r (combination)
  $$^nC_r = \frac{n!}{r!\,(n-r)!}$$
  $^nC_n = 1$

- **Binomial Distribution**
  If an experiment consists of n independent trials, and for each trial, the probability than an event occurs is P(E) = p whilst the probability that is does not occur is $P(\tilde{E})$ = q where q = 1 – p then the respective probabilities of the event E occurring exactly 0, 1, 2, 3, 4, ... , r, ..., n times are the terms in ascending powers of p, in the binomial expansion of $(q + p)^n$.

  The probability than the event E will occur exactly r times and thus fail (n – r) times in the n independent trials is:

  $$P(R\text{ successes}) = {}^nC_r\, q^{n-r}\, p^r$$

- **Expected Value**
  The expectation of X or the expected value of X is defined as: E(X) = ΣX . P(X)

  If the random variable of X donates the number of times a certain event occurs is *n* independent trials, then the expected value occurs in *n* independent trials, then the expected value of X is given by: E(X) = np, where p is the probability of that event occurring in each trial.