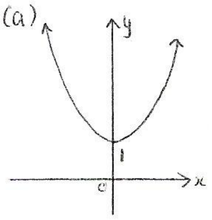
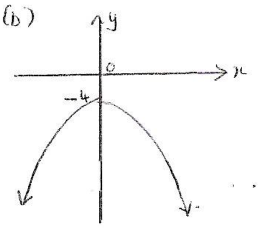
QUADRATIC SOLUTIONS

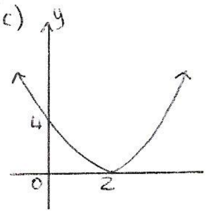
1 (a)



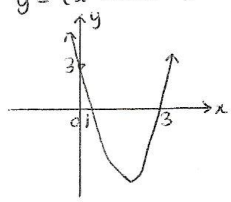
(b)



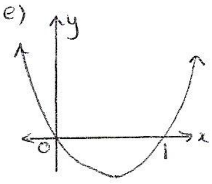
(c)



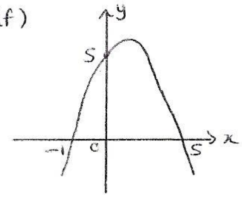
(d) $y = (x-3)(x-1)$



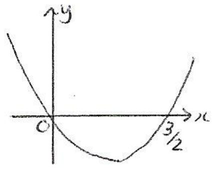
(e)



(f)

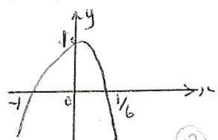


(g) $y = x(2x-3)$



(h) $y = 1-5x-6x^2$

$y = (1+x)(1-6x)$



8

2 (a) $x^2-9x+14<0$

$(x-7)(x-2)<0$

$2<x<7$

(b) $x^2+7x>0$

$x(x+7)>0$

$x<-7$ or $x>0$

(c) $4x-x^2\leq 0$

$x^2-4x\geq 0$

$x(x-4)\geq 0$

$x\leq 0$ or $x\geq 4$

(d) $12+4x-x^2\geq 0$

$x^2-4x-12\leq 0$

$(x-6)(x+2)\leq 0$

$-2\leq x\leq 6$ 8

3 The general quadratic is

$x^2-(\alpha+\beta)x+\alpha\beta=0$

(a) $\alpha+\beta=5$ and $\alpha\beta=0$

$\therefore x^2-5x+6=0$ 1

(b) $\alpha+\beta=-5/3$ $\alpha\beta=-2/3$

$\therefore x^2+\frac{5}{3}x-\frac{2}{3}=0$ 1

$3x^2+5x-2=0$

(c) $\alpha+\beta=2-\sqrt{3}+2+\sqrt{3}=4$

$\alpha\beta=(2-\sqrt{3})(2+\sqrt{3})=4-3=1$ 2 4

$\therefore x^2-4x+1=0$

4 $x^2-4x-3=0$

(a) $\alpha+\beta=-{}^{-4}/_{1}=4$ 1

(b) $\alpha\beta=-3/1=-3$ 1

(c) $(\alpha+4)(\beta+4)=\alpha\beta+4(\alpha+\beta)+16$

$=-3+4(4)+16$ 2

$=29$

(d) $\alpha^2\beta+\alpha\beta^2=\alpha\beta(\alpha+\beta)$

$=-3\times 4$ 2

$=-12$

(e) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=4^2-2(-3)$

$=16+6$ 2

$=22$

(f) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\beta+\alpha}{\alpha\beta}=4\div -3=-\frac{4}{3}$ 2

(g) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{22}{-3}$ 2

(h) $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2=\alpha^2+\beta^2-2\alpha\beta$

$=22-2(-3)$

$=28$ 14 2

5 When $x=-1$, $a(-1)^2-b(-1)-10=0$

$a+b=10 \ldots (1)$ 1

when $x=5$, $a(5)^2-b(5)-10=0$

$25a-5b=10$

$5a-b=2 \ldots (2)$ 1

(2) + (1) $6a=12$

$a=2$

subs in (1) $2+b=10 \therefore b=8$

$\therefore a=2, b=8$ 1 3

6 Let the roots be α and 3α

sum of roots: $\alpha+3\alpha=-q/1$

$\therefore 4\alpha=-q$

$\alpha=-q/4 \ldots (1)$ 1

product of roots: $\alpha\cdot 3\alpha=r/1$

$\therefore 3\alpha^2=r \ldots (2)$ 1

subs (1) into (2) $3\left(\frac{-q}{4}\right)^2=r$

$\frac{3q^2}{16}=r$

$\therefore 3q^2=16r$ 1 3

7 $x^2-(k+2)x+4k=0$

(a) $-b/a=5 \therefore \frac{k+2}{1}=5$

$k+2=5$

$k=3$ 1

(b) $c/a=12 \therefore \frac{4k}{1}=12$

$k=3$ 1

(c) Let the roots be α and $\alpha+2$

sum of roots: $\alpha+\alpha+2 = \frac{k+2}{1}$

$2\alpha+2 = k+2$

$2\alpha = k$

$\alpha = k/2 \quad \ldots (1)$

product of roots: $\alpha(\alpha+2) = \frac{4k}{1}$

$\alpha^2+2\alpha = 4k \quad \ldots (2)$

subs (1) into (2) $\left(\frac{k}{2}\right)^2 + 2\left(\frac{k}{2}\right) = 4k$

$\frac{k^2}{4} + k = 4k$

$k^2 + 4k = 16k$

$k^2 - 12k = 0$

$k(k-12) = 0$

$\therefore k = 0, 12$

5

(8) Prove that $\Delta < 0$ and $a > 0$

$\Delta = (-4)^2 - 4\times2\times5 \qquad a = 2$

$= 16 - 40 \qquad \therefore a > 2$

$= -24$

$\therefore \Delta < 0$

Hence, expression is pos. def.

2

(9) One root exists when $\Delta = 0$

$\therefore (-4m)^2 - 4(5m-3)(m+1) = 0$

$16m^2 - 4(5m^2+2m-3) = 0$

$16m^2 - 20m^2 - 8m + 12 = 0$

$-4m^2 - 8m + 12 = 0$

$m^2 + 2m - 3 = 0$

$(m+3)(m-1) = 0$

$m = -3, 1$

2

(10) For roots to be rational Δ must be a perfect square.

$\Delta = [-(a+b)]^2 - 4a.b$

$= a^2+2ab+b^2-4ab$

$= a^2-2ab+b^2$

$= (a-b)^2$

Hence, roots are rational

2

(11) $x^2+10x+10 = a(x+2)^2+b(x+1)$

$= a(x^2+4x+4)+bx+b$

$= ax^2+4ax+4a+bx+b$

Equating coefficients gives

$a = 1 \quad \ldots (1)$

$4a+b = 10 \quad \ldots (2)$

$\therefore 4+b = 10$

$b = 6$

Hence, $x^2+10x+10 = 1(x+2)^2+6(x+1)$

3

(12) max value $= 6-5(0) = 6$

which occurs when $x+2=0$

i.e. $x = -2$

2

(13) $y = 5+6x-x^2$

(a) $x = -b/2a$

$= -6/2\times-1$

$\therefore x = 3$

(b) when $x = 3$, $y = 5+6(3)-3^2$

$= 14$

$\therefore$ vertex is $(3, 14)$

2

(14) (a) Let $u = x^2 \therefore u^2-10u+9=0$

$(u-9)(u-1)=0$

$u = 1, 9$

when $u=1$, $x^2=1 \therefore x=\pm1$

when $u=9$, $x^2=9 \therefore x=\pm3$

$\therefore x = \pm1, \pm3$

(b) Let $u = x^2-2x \therefore u^2-11u+24=0$

$(u-3)(u-8)=0$

$u = 3, 8$

when $u=3$, $x^2-2x=3$

$x^2-2x-3=0$

$(x-3)(x+1)=0$

$x = 3, -1$

when $u=8$, $x^2-2x=8$

$x^2-2x-8=0$

$(x-4)(x+2)=0$

$x = 4, -2$

$\therefore x = -2, -1, 3, 4$

(c) Let $u = 2^x \therefore u^2-3u+2=0$

$(u-2)(u-1)=0$

$u = 1, 2$

when $u=1$, $2^x = 1$

$2^0 = 1$

$\therefore x = 0$

when $u=2$, $2^x = 2 \therefore x = 1$

$\therefore x = 0, 1$

9

(15) $y = mx-6 \quad \ldots (1)$

$y = x^2-2x+3 \quad \ldots (2)$

Equate (1) and (2) $x^2-2x+3 = mx-6$

$x^2-2x-mx+9=0$

$x^2-(2+m)x+9=0$

line touches parabola $\therefore$ there should be only one solution. Hence, $\Delta = 0$

$(2+m)^2-4\times1\times9 = 0$

$4+4m+m^2-36 = 0$

$m^2+4m-32=0$

$(m+8)(m-4)=0 \quad \therefore m = -8, 4$

3