ANSWERS

(1) $a+4d=-17 \quad \ldots (1)$
$a+8d=-29 \quad \ldots (2)$
(2) − (1) $\quad 4d=-12$
$d=-3$
subs in (1) $\quad a-12=-17$
$a=-5$.
∴ sequence is $-5, -8, -11, \ldots$ (3)

(2) $a=25$
$a+6d=7$
∴ $25+6d=7$
$6d=-18$
$d=-3$
∴ sequence is 25, 22, 19, 16, 13, 10, 7 (3)

(3) $a=4$, $d=3$
$S_n=531$ ∴ $\frac{n}{2}[8+(n-1).3]=531$
$n(8+3n-3)=1062$.
$n(5+3n)=1062$
$3n^2+5n-1062=0$
$n=\frac{-5\pm\sqrt{25-4.3.-1062}}{6}$
$=\frac{-5\pm 113}{6}$
$=18, \frac{-118}{6}$
But $n>0$ ∴ 18 terms are required (3)

(4) $a+9d=39 \ldots (1)$ ie $U_{10}=39$.
$S_{10}=165$ ∴ $\frac{10}{2}[a+39]=165$
∴ $5a+195=165$
$5a=-30$
$a=-6$.
subs in (1) gives $-6+9d=39$
$9d=45$
$d=5$
∴ $S_{20}=\frac{20}{2}[-12+19\times 5]$.
$=10\times 83$
$=830$. (4)

(5) $S_n=2n^2+n$
$S_{n-1}=2(n-1)^2+(n-1)$
$=2(n^2-2n+1)+n-1$
$=2n^2-4n+2+n-1=2n^2-3n+1$
$U_n=S_n-S_{n-1}$
$=2n^2+n-(2n^2-3n+1)$
$=2n^2+n-2n^2+3n-1$
$=4n-1$ (3)

(6) (a) $\sum_{k=1}^{8}(1+4k)=5+9+13+\ldots+33$
$=\frac{8}{2}[5+33]$
$=152$. (2)
(b) $\sum_{k=1}^{9}k^2$ (3)

(7) $U_1=a=2 \quad \ldots (1)$
$U_5=ar^4=162 \quad \ldots (2)$
(2) ÷ (1) $\quad r^4=81$ ∴ $r=\pm 3$
∴ sequence is 2, 6, 18, 54, 162 or
2, −6, 18, −54, 162 (3)

(8) $a=3$, $r=2$.
$S_8=\frac{3(2^8-1)}{2-1}=765$ (2)

(9) $a=4$, $r=\frac{1}{2}$
$S_{10}=\frac{4\left(1-\frac{1}{2}^{10}\right)}{1-\frac{1}{2}}=4\times\frac{1023}{1024}\times 2$.
$=\frac{8184}{1024}$
$=\frac{1023}{128}$ (2)

(10) $a=8 \quad S_\infty=\frac{8}{1-r}=32$
$32-32r=8$
$32r=24$
$r=\frac{24}{32}=\frac{3}{4}$ (2)

(11) $0.5\dot{7}=0.577777\ldots$
$=0.5+\{0.07+0.007+\ldots\}$
$=\frac{1}{2}+\frac{0.07}{1-0.1}=\frac{1}{2}+\frac{0.07}{.90}=\frac{1}{2}+\frac{7}{90}$
$=\frac{26}{45}$
∴ $0.5\dot{7}=\frac{26}{45}$ (3)

(12) (a) $A=500(1+0.08)^{10}$
$=500(1.08)^{10}$
$=\$1079.46$
(b) Let A_n = be accumulated amount of each investment.
∴ $A_1=500(1.08)^{10}$

$A_{2} = 500(1.08)^9$

⋮

$A_{10} = 500(1.08)^1$

∴ Total investment is worth

$= 500(1.08) + 500(1.08)^2 + \dots + 500(1.08)^{10}$

$= 500(1.08)[1 + (1.08) + \dots + (1.08)^9]$.

$= 500(1.08) \cdot \frac{1[(1.08)^{10}-1]}{1.08-1}$ 3

$= \frac{500(1.08)[1.08^{10}-1]}{0.08}$

$= \$7822.74$ 4

13 Let A_n = amount owing after each instalment

∴ $A_1 = 5000(1.015)^1 - 100$

$A_2 = [5000(1.015)^1 - 100](1.015) - 100$

$= 5000(1.015)^2 - 100(1.015) - 100$

$= 5000(1.015)^2 - 100(1 + 1.015)$

⋮

$A_8 = 5000(1.015)^8 - 100(1 + 1.015 + 1.015^2 + \dots + 1.015^7)$

$= 5000(1.015)^8 - \frac{100.1(1.015^8-1)}{1.015-1}$

$= 5000(1.015)^8 - \frac{100(1.015^8-1)}{0.015}$ 3

$= \$4789.18$

14 A_n = amount owing

$A_1 = 5000(1.04) - M$

$A_2 = [5000(1.04) - M](1.04) - M$

$= 5000(1.04)^2 - M(1+1.04)$

⋮

$A_{10} = 5000(1.04)^{10} - M(1+1.04+\dots+1.04^9)$

But at end of this time $A_{10} = 0$

∴ $5000(1.04)^{10} - \frac{M(1.04^{10}-1)}{0.04} = 0$

$\frac{M(1.04^{10}-1)}{0.04} = 5000(1.04)^{10}$ 3

∴ $m = \frac{5000(0.04)(1.04)^{10}}{1.04^{10}-1}$

$= \$616.45$

15 $a = 1$, $r = 2x$ and $S_\infty = \frac{3}{4}$

∴ $\frac{1}{1-2x} = \frac{3}{4}$

$3 - 6x = 4$

$6x = -1$ 2

$x = -\frac{1}{6}$

16 $\frac{2m+4}{2m-8} = \frac{5m-2}{2m+4}$

$(2m+4)^2 = (2m-8)(5m-2)$

$4m^2+16m+16 = 10m^2-4m-40m+16$.

$6m^2 - 60m = 0$ 2

$6m(m-10) = 0$

∴ $m = 0, 10$

17 Amount $= 256000(0.75)^4$ 1

$= 81000$ L

18 $a = 30$, $d = -4$, $S_n = 120$

∴ $\frac{n}{2}[60 + (n-1).-4] = 120$

$\frac{n}{2}[60 - 4n + 4] = 120$

$n(64-4n) = 240$.

$64n - 4n^2 = 240$

$4n^2 - 64n + 240 = 0$ 3

$n^2 - 16n + 60 = 0$

$(n-6)(n-10) = 0$

∴ either $n = 6$ or 10

19 15, 16, 17, ...

$a = 15$, $d = 1$, $S_n = 246$.

∴ $\frac{n}{2}[30 + (n-1).1] = 246$.

$\frac{n}{2}[29 + n] = 246$

$n(29+n) = 492$

$n^2 + 29n - 492 = 0$

$n = \frac{-29 \pm \sqrt{29^2 - 4.1.-492}}{2}$

$= \frac{-29 \pm 53}{2}$

$= 12, -41$ 2

but since $n > 0$ then $n = 12$

ie 12 rows

(a) $U_{12} = 15 + 11 \times 1 = 26$ 3

∴ 26 logs. 1