REVISION QUESTIONS

The Parabola (and Circle)

1. For each parabola, write down the vertex, focus, directrix and hence sketch the curve.

   (a) $x^2 = 12$ (b) $y^2 = x$ (c) $y^2 = -8x$ (d) $x^2 = -4y$

2. For each parabola, find the coordinates of the focus and vertex, the equation of the axis and directrix and sketch their graphs.

   (a) $(x + 2)^2 = -12(y + 3)$ (b) $(y - 1)^2 = 6(x - 4)$

3. By completing the square, find the coordinates of the focus and the equation of the directrix of each parabola.

   (a) $y = x^2 - 8x + 2$ (b) $x = y^2 + 4y - 10$

4. Find the equation of the parabola with:

   (a) vertex (3, –2) and focus (3, 7) (b) vertex (–1, 5) and directrix $y = 8$

   (c) focus (2, 4) and directrix $x = -2$ (d) passing through (1, 5), (–1, 9) and (0, 4).

5. Find the equation of the locus of the points P(x, y):

   (a) which are equidistant from A(–4, 1) and B(7, –2)

   (b) whose distance from the point Q(11, –11) is twice its distant from R(2, 1)

   (c) whose distance from the point S(6, –7) is equal to their distance from the line $y = -9$

   (d) whose distance from the point (1, –5) is equal to 4 units

   (e) such that PQ is perpendicular to PR where Q is (1, 5) and R is (9, –1)

   (f) such that $PA^2 + PB^2 = 100$, where A and B are the points (1, 5) and (7, 5) respectively.

6. $(x + 1)^2 + (y - 3)^2 = 4$ represents a circle.

   (a) Sketch the circle clearly marking the centre and the radius.

   (b) Show that the line $3x - 4y + 5 = 0$ is a tangent to the circle $(x + 1)^2 + (y - 3)^2 = 4$.

7. Find the equation of the tangent to the parabola $x^2 = 16y$ at the point (12, 9) on the curve.

Geometrical Applications of the Derivative

1. Sketch the curves $y = f(x)$ satisfying the following conditions:

   (a) $f(0) = 1$; $f'(x) > 0$ for $0 \le x < 3$; $f'(3) = 0$; $f(3) = 5$; $f'(x) < 0$ for $x > 0$; $f(4) = 0$

(b) $f(-3) = 12$; $f(0) = 6$; $f(3) = 0$; $f'(-3) = f'(3) = 0$
$f'(x) < 0$ for $-3 < x < 3$; $f'(x) > 0$ for $x > 3$ or $x < -3$
$f''(x) < 0$ for $x < 0$; $f''(x) > 0$ for $x > 0$

2. For what values x is the function $f(x) = x^2 + 8x - 5$ monotonic decreasing?

3. Show that the curve $y = 2x^3 + 6x - 1$ is always increasing.

4. Find the stationary points on the curve $y = 3x^4 + 8x^3 - 18x^2 + 1$.

5. Find the stationary points on the curve $y = 3x^4 - 4x^3 + 7$ and determine the nature.

6. Find the second derivative of $f(x) = 2x^6 - 3x^4 + 9x^3 - 5x^2 + 7x + 1$.

7. Find $\dfrac{d^2x}{dy^2}$ if $y = \dfrac{7}{x} + \sqrt[3]{x^2} + x^3$.

8. Find the second derivative of $y = 3x(x-2)^7$.

9. Find $f''(-1)$ if $f(x) = 2x^3 - 2x^4$.

10. For what values of x is the curve $y = 2x^3 + 12x^2 - 17x + 15$ concave up?

11. (a) Show that the curve $y = x^3 - 6x^2 + 12x + 8$ has a point of inflexion at $x = 2$

(b) Is it a horizontal point of inflexion?

12. For the curve $y = 3x^2 - x^3$

13. (a) determine where the curve meets the coordinate axes

(b) find all stationary points and determine their nature

(c) find any points of inflexion

(d) determine the nature of the curve for very large positive and negative values of the curve.

(e) sketch the curve.

<u>ANSWERS</u>

<u>The Parabola</u>

1. (a) vertex = (0, 0); focus = (0, 3) ; directrix is $y = -3$

(b) vertex = (0, 0); focus = $(\frac{1}{4}, 0)$; directrix is $y = -\frac{1}{4}$

(c) vertex = (0, 0); focus = (0, –2) ; directrix is $y = 2$

(d) vertex = (0, 0); focus = (–1, 0) ; directrix is $x = 1$

2. (a) vertex = (–2, –3); focus = (–2, 0); axis is $x = -2$; directrix is $y = -6$

(b) vertex = (4, 1); focus = (5.5, 1); axis is $y = 1$; directrix is $y = 2.5$

3. (a) focus is (4, –13.75) and directrix is $y = -14.25$

(b) focus is (–1.75, –14) and directrix is $x = -2.25$

4. (a) $(x-3)^2 = 36(y+2)$ (b) $y = (x+1)^2 = -12(y-5)$

(c) $(y-4)^2 = 8x$ (d) $y = 3x^2 - 2x + 4$

5. (a) $11x - 3y = 18$ (b) $3x^2 + 3y^2 + 6x - 30y - 222 = 0$

(c) $x^2 = 12x + 4y - 4$ (d) $(x-1)^2 + (y-1)^2 = 16$

(e) $x^2 + y^2 - 10x - 6y + 14 = 0$ (f) $x^2 - 8x + y^2 - 2y - 8 = 0$

6. Centre (–1, 3) and radius 2 units.

7. $3x - 2y - 18 = 0$

Geometric Applications of the Derivative

2. $x < -4$

4. (0, 1) ; (1, –6) and (–3, –134)

5. (0, 7) is a horizontal point of inflexion and (1, 6) is a minimum turning point.

6. $y'' = 60x^4 - 36x^2 + 54x - 10$

7. $\dfrac{d^2x}{dy^2} = \dfrac{14}{x^3} - \dfrac{2}{9\sqrt[3]{x^4}}$

8. $y'' = 84(x-1)^5(2x-1)$

9. –48

10. $x > -2$

11. (b) yes

12. (a) (0, 0) and (3, 0)

(b) (0, 0) is a minimum turning point and (2, 4) is a maximum turning point

(c) (1, 2) is a point of inflexion

(d) the curve tends to positive and negative infinity