TIONS

Mathematics

(a) $\sin 28° = \frac{x}{5}$ $\therefore x = 5\sin 28°$

$= 2.34735\ldots$

$\doteqdot 2.35m$ 2

(b) $\cos 67°14' = \frac{x}{8.1}$ $\therefore x = 8.1\cos 67°14'$

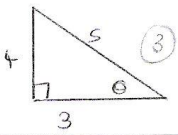
$= 3.1345\ldots$

$= 3.13km$ 2 (4)

(2) $\tan\theta = 5/7$ $\theta = 35°32'$ (2)

(3) (a) $\sin\theta = \pm 4/5$

(b) $\cot\theta = 3/4$



(3)

(4) (a) $\tan 225° = \tan(180° + 45°)$

$= \tan 45°$

$= 1$

(b) $\sin 135° = \sin(180° - 45°)$

$= \sin 45°$

$= \frac{1}{\sqrt{2}}$

(c) $\cos 300° = \cos(360° - 60°)$

$= \cos 60°$

$= \frac{1}{2}$

(d) $\cos 240° = \cos(180° + 60°)$

$= -\cos 60°$

$= -\frac{1}{2}$

(e) $\tan 480° = \tan(360° + 120°)$

$= \tan 120°$

$= -\tan(180° - 60°)$

$= -\tan 60°$

$= -\sqrt{3}$

(f) $\sin(-45°) = \sin 315°$

$= \sin(360° - 45°)$

$= -\sin 45°$

$= -\frac{1}{\sqrt{2}}$ (12)

(5) (a) LHS $= \operatorname{cosec}\theta - \sin\theta$

$= \frac{1}{\sin\theta} - \sin\theta$

$= \frac{1 - \sin^2\theta}{\sin\theta}$

$= \frac{\cos^2\theta}{\sin\theta}$

$= \frac{\cos\theta \cdot \cos\theta}{\sin\theta}$ 3

$= \cot\theta \cdot \cos\theta$

$=$ RHS

(b) LHS $= \frac{\tan\theta}{\sec\theta - 1} - \frac{\tan\theta}{\sec\theta + 1}$

$= \frac{\tan\theta(\sec\theta + 1) - \tan\theta(\sec\theta - 1)}{(\sec\theta - 1)(\sec\theta + 1)}$

$= \frac{\tan\theta\sec\theta + \tan\theta - \tan\theta\sec\theta + \tan\theta}{\sec^2\theta - 1}$

$= \frac{2\tan\theta}{\tan^2\theta}$

$= \frac{2}{\tan\theta}$

$= 2\cot\theta$ 3

$=$ RHS (6)

(6) $12\cos\theta = 7\cos\theta + 2$

$5\cos\theta = 2$

$\cos\theta = 2/5$

Basic angle is $66°25'$

$\therefore \theta = 66°25', 360° - 66°25'$

$= 66°25', 293°35'$ (2)

(7) (a) $\frac{x}{\sin 98°} = \frac{7}{\sin 51°}$

$x = \frac{7\sin 98°}{\sin 51°}$

$= 8.9196\ldots$

$\approx 8.9m$

(b) $x^2 = 4.7^2 + 11.3^2 - 2\times 4.7\times 11.3\times\cos 56°$

$= 90.382529\ldots$

$x = 9.5069\ldots$

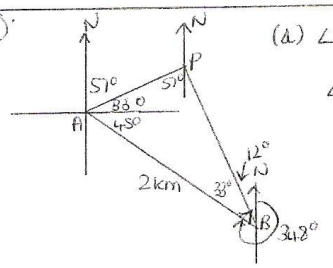
$= 9.5m$ (4)

(8) Area $= \frac{1}{2}\times 4.7\times 11.3\times\sin 56°$

$= 22.015\ldots$

$= 22.0m^2$ (1 dec. pl.) (2)

(9)



(a) $\angle PAB = 33° + 45°$

$= 78°$ 1

$\angle APB = 57° + 12°$

$= 69°$ 1

(b) $\frac{AP}{\sin 33°} = \frac{2}{\sin 69°}$

$AP = \frac{2\sin 33°}{\sin 69°}$ 2 (4)

$= 1.16677\ldots$

$\doteqdot 1.17$ Km

Extension 1

① (a) LHS $= \dfrac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$

$= \dfrac{\sin A\cos B+\cancel{\sin B\cos A}+\sin A\cos B-\cancel{\sin B\cos A}}{\cos A\cos B-\cancel{\sin A\sin B}+\cos A\cos B+\cancel{\sin A\sin B}}$

$= \dfrac{2\sin A\cancel{\cos B}}{2\cos A\cancel{\cos B}}$

$= \dfrac{\sin A}{\cos A}$

$= \tan A$ 3

$= RHS$

(b) LHS $= \dfrac{\sin 2A+\sin A}{1+\cos 2A+\cos A}$

$= \dfrac{2\sin A\cos A+\sin A}{1+\cos^2 A-\sin^2 A+\cos A}$

$= \dfrac{\sin A(2\cos A+1)}{2\cos^2 A+\cos A}$

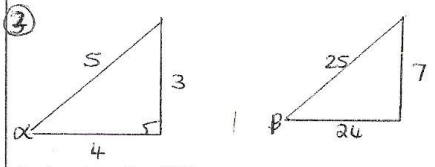
$= \dfrac{\sin A\cancel{(2\cos A+1)}}{\cos A\cancel{(2\cos A+1)}}$

$= \dfrac{\sin A}{\cos A}$ 3 (6)

$= \tan A$

$= RHS$

③



1 1

(a) $\sin(\alpha-\beta)$

$= \sin\alpha\cos\beta-\sin\beta\cos\alpha$

$= \frac{3}{5}\cdot\frac{24}{25}-\frac{7}{25}\cdot\frac{4}{5}$

$= \frac{44}{125}$ 2

(b) $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$= \dfrac{\frac{3}{4}+\frac{7}{24}}{1-\frac{3}{4}\cdot\frac{7}{24}}$

$= \frac{25}{24}\div\frac{25}{32}$ 2

$= \frac{4}{3}$

(c) $\cos 2\alpha = \cos^2\alpha-\sin^2\alpha$

$= \frac{16}{25}-\frac{9}{25}$ 2 (8)

$= \frac{7}{25}$

④ (a) $\sin\theta+\cos\theta = \dfrac{2t}{1+t^2}+\dfrac{1-t^2}{1+t^2}$

$= \dfrac{2t+1-t^2}{1+t^2}$ 2

(b) $\dfrac{2}{1+\cos\theta} = \dfrac{2}{1+\frac{1-t^2}{1+t^2}} = 2\div\dfrac{1+t^2+1-t^2}{1+t^2}$

$= 2\times\dfrac{1+t^2}{2}$ 2 (4)

$= 1+t^2$

⑤ (a) $\dfrac{1}{\tan\theta}-3\tan\theta = 2$ $\therefore 1-3\tan^2\theta = 2\tan\theta$

$3\tan^2\theta+2\tan\theta-1=0$

$(3\tan\theta-1)(\tan\theta+1)=0$

$\therefore \tan\theta = \frac{1}{3}$ or -1 2

when $\tan\theta=\frac{1}{3}$, basic angle is $18°26'$

$\therefore \theta = 18°26', 180°+18°26'$ 1

$= 18°26', 198°26'$

when $\tan\theta=-1$, basic angle is $45°$

$\therefore \theta = 180°-45°, 360°-45°$ 1

$= 135°, 315°$

$\therefore \theta = 18°26', 135°, 198°26', 315°$

(b) $2\sin\theta\cos\theta = \sin\theta$

$2\sin\theta\cos\theta-\sin\theta=0$

$\sin\theta(2\cos\theta-1)=0$ 2

$\therefore \sin\theta=0$ or $\cos\theta=\frac{1}{2}$

when $\sin\theta=0$, $\theta=0°, 180°, 360°$ 1

when $\cos\theta=\frac{1}{2}$, basic angle is $60°$

$\therefore \theta = 60°, 360°-60°$ 1

$= 60°, 300°$

$\therefore \theta = 0°, 60°, 180°, 300°, 360°$

(c) $\sin^2\theta-5\sin\theta-2(1-\sin^2\theta)=0$

$\sin^2\theta-5\sin\theta-2+2\sin^2\theta=0$

$3\sin^2\theta-5\sin\theta-2=0$

$(3\sin\theta+1)(\sin\theta-2)=0$ 2

$\therefore \sin\theta = -\frac{1}{3}$ or 2

when $\sin\theta=-\frac{1}{3}$, basic angle is $19°38'$

$\therefore \theta = 180°+19°38', 360°-19°38'$

$= 199°38', 340°22'$ 1

when $\sin\theta=2$ there is no solution (12)

$\therefore \theta = 199°38', 340°22'$ 1

⑥ (a) $R = \sqrt{(\sqrt{3})^2+1^2} = \sqrt{4} = 2$

$\tan\alpha = \frac{1}{\sqrt{3}}$ $\therefore \alpha = 30°$ 2

$\therefore \sqrt{3}\sin\theta-\cos\theta = 2\sin(\theta-30°)$

(b) $2\sin(\theta-30°)=1$

$\sin(\theta-30°)=\frac{1}{2}$

Basic angle is $30°$

$\therefore \theta-30° = 30°, 150°$ 2 (4)

$\theta = 60°, 180°$

(a) $\tan 29^\circ = \dfrac{2000}{AE}$

$\therefore AE = \dfrac{2000}{\tan 29^\circ}$

$= 3608.095511\ldots$

$\approx 3608\text{ m}$ 2

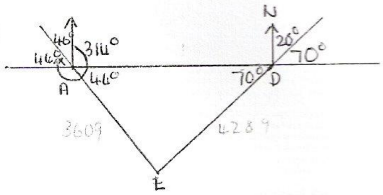
(b) $\tan 25^\circ = \dfrac{2000}{DE}$

$DE = \dfrac{2000}{\tan 25^\circ}$

$= 4289.0138\ldots$ 2

$\approx 4289\text{ m}$

(c)



$\angle AED = 180^\circ - 46^\circ - 70^\circ$

$= 66^\circ$ 1

(d) $d^2 = 3608^2 + 4289^2 - 2 \times 3608 \times 4289 \cos 66^\circ$

$= 18824920.18\ldots$

$d = 4338.769\ldots$ 2

$\therefore$ distance travelled is approx 4339m

Speed $= 4339\text{ m/min}$

$= \dfrac{4339 \times 60}{1000}\text{ km/h}$ 2

$= 260.34\text{ km/h}$.

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(2) $\cos(105^\circ) = \cos(60^\circ + 45^\circ)$ 1

$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$ 1

$= \dfrac{1}{2} \cdot \dfrac{1}{\sqrt{2}} - \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{\sqrt{2}}$

$= \dfrac{1 - \sqrt{3}}{2\sqrt{2}}$

$= \dfrac{\sqrt{2} - \sqrt{6}}{4}$ 1

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