MATHEMATICS TRIGONOMETRY SUMMARY

1. Basic Trigonometric Ratios

$\sin\theta = \dfrac{opposite}{hypotenuse}$ $\cos\theta = \dfrac{adjacent}{hypotenuse}$ $\tan\theta = \dfrac{opposite}{adjacent}$

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

2. Complementary Ratios

$\sin(90° - \theta) = \cos\theta$ $\cos(90° - \theta) = \sin\theta$ $\tan(90° - \theta) = \cot\theta$

3. Pythagorean Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$ **

$\sin^2\theta = 1 - \cos^2\theta$ (on rearranging **)

$\cos^2\theta = 1 - \sin^2\theta$ (on rearranging **)

$1 + \tan^2\theta = \sec^2\theta$ (on dividing ** by $\cos^2\theta$)

$1 + \cot^2\theta = \text{cosec}^2\theta$ (on dividing ** by $\sin^2\theta$)

4. The Sine Rule

This is used in triangles which are not right-angled. It is used when given two sides and two angles, one of which is unknown.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

5. The Cosine Rule

This is used in triangles which are not right-angled. It is used when given three sides and one angle, one of which is unknown.

To fine a side: $a^2 = b^2 + c^2 - 2bcCosA$

To find an angle: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

6. The Area of a Triangle

This is used in triangles which are not right-angled. It is used when given two sides and the included angle.

$$\text{Area} = \frac{1}{2}ab\sin C$$

EXTENSION 1 TRIGONOMETRY SUMMARY

1. Compound Angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\cos(A - B) = \cos A\cos B + \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

2. Double Angle Formulae

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 1 - 2\sin^2 A$ and on rearranging this gives: $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

$\cos 2A = 2\cos^2 A - 1$ and on rearranging this gives: $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

3. The 't' Formulae

Given that $t = \tan\left(\dfrac{x}{2}\right)$:

$\sin x = \dfrac{2t}{1 + t^2}$ $\cos x = \dfrac{1 - t^2}{1 + t^2}$ $\tan x = \dfrac{2t}{1 - t^2}$

4. Subsidiary Angle Method

Angles of the form $a\sin\theta + b\cos\theta = c$ can be solved using the subsidiary method.

They are written in the form $R\sin(\theta + \alpha)$ where:

$R = \sqrt{a^2 + b^2}$ and $\tan\alpha = \dfrac{a}{b}$ and α is acute.

Note: There are different forms of the original equation,

5. General Solution for Trigonometric Equations

If $\sin\theta = \sin\alpha$ then the general solution is $\theta = 180n + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then the general solution is $\theta = 360n \pm \alpha$

If $\tan\theta = \tan\alpha$ then the general solution is $\theta = 180n + \alpha$