DAPTO HIGH SHCOOL

YEAR 11

EXTENSION 1 MATHEMATICS

PRELIMINARY EXAMINATION

2006

Time Allowed – 2 hours
(plus 5 minutes reading time)

Directions:

- **Attempt ALL questions**
- **All questions are of equal value**
- **All necessary working should be shown**
- **Board-approved calculators may be used**

Marks

Question 1 (12 marks)

(a) Differentiate with respect to x

(i) $\left(x^2 - 5x\right)^4$ **2**

(ii) $\dfrac{2x-3}{5-x}$ **2**

(iii) x^{n^2} **1**

(iv) $(x^2 - 5x + 7)(3x + 2)$ **2**

(b) Given the function $y = 10x^3 - 4$. Find the value(s) of x for which $\dfrac{dy}{dx} = 10$. **2**

(c) Find the gradient of the normal to the curve $y = \dfrac{1}{\sqrt{x^2 - 3}}$ at the point (2, 1) **3**

Question 2 (12 marks)

(a) If α and β are the roots of the quadratic equation $2x^2 - 3x - 1 = 0$. Evaluate

(i) $\alpha + \beta$ **1**

(ii) $\dfrac{1}{\alpha\beta}$ **1**

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(b) Given that $x^2 + 7x - 3 \equiv a(x + 1)^2 + b(x + 1) + c$, find the values of a, b and c **3**

(c) Show that for $kx^2 + 21x + m = 0$, the roots are equal if $l^2 = km$ **2**

(d) Solve $x^6 + 26x^2 - 27 = 0$ **3**

Marks

Question 3 (12 marks)

(a) For the parabola $x^2 = 10y$, find

(i) the coordinates of the focus **1**

(ii) the equation of the directrix **1**

(iii) the length of the latus rectum **1**

(b) Find the coordinates of the focus of the parabola $x^2 = 8(y - 1)$ **2**

(c) A curve has parametric equations $x = \frac{t}{2}$, $y = 3t^2$. Find the cartesian equation for this curve. **2**

(d) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.
The equation of the normal to the parabola at P is $x + py = 2ap + ap^3$ and the equation of the normal at Q is similarly given by $x + qy = 2aq + aq^3$.

(i) Show that the normals at P and Q intersect at the point R whose coordinates are $\left(-apq[p + q], a[p^2 + pq + q^2 + 2]\right)$. **2**

(ii) The equation of the chord PQ is $y = \frac{1}{2}(p + q)x - apq$. (Do NOT show this. If the chord PQ passes through $(0, a)$ show that $pq = -1$. **1**

(iii) Find the equation of the locus of R if the chord PQ passes through $(0, a)$. **2**

Marks

Question 4 (12 marks)

(a) Solve $\frac{3}{x-2} \leq 1$ **3**

(b) Find the coordinates of the point P that divides the interval joing $(-3, 4)$ and $(5, 6)$ internally in the ratio $1 : 3$. **2**

(c) The acute angle between the lines $y = 3x + 5$ and $y = mx + 4$ is 45°. Find the two possible values of m. **2**

(d) Let $f(x) = \frac{x}{x^2 - 1}$

(i) For what values of x is $f(x)$ undefined? **2**

(ii) Show that $y = f(x)$ is an odd function **1**

(iii) Sketch $y = f(x)$ **2**

Question 5 (12 marks)

(a) The polynomial $P(x) = x^3 - 2x^2 + a$ has a remainder of 3 when divided by $(x + 2)$. Find the value of a. **2**

(b) It is known that two of the roots of the equation $2x^3 + x^2 - kx + 6 = 0$ are reciprocals of each other. Find the value of k. **2**

(c) The polynomial $P(x) = x^3 - 2x^2 + kx + 24$ has roots α, β, γ .

(i) Find the value of $\alpha + \beta + \gamma$ **1**

(ii) Find the value of $\alpha\beta\gamma$ **1**

(iii) It is known that two of the roots are equal in magnitude by opposite in sign. Find the third root and hence find the value of k. **2**

(d) A polynomial is given by $P(x) = x^3 + ax^2 + bx - 18$. Find the values for a and b if $(x + 2)$ is a factor of $P(x)$ and if -24 is a remainder when $P(x)$ is divided by $(x - 1)$. **4**

Marks

Question 6 (12 marks)

(a) Solve $\sin 2\theta = -\frac{1}{2}$ for $0° \leq \theta \leq 360°$ **2**

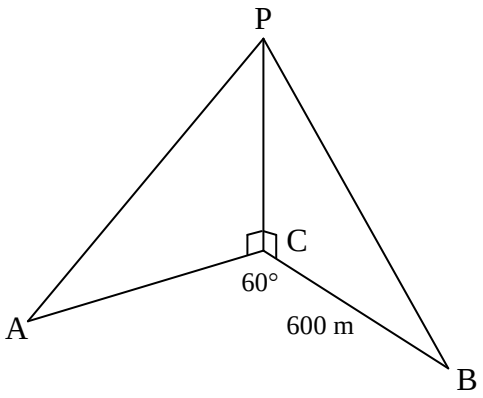
(b) By expanding the left hand side, show that **1**
$\sin(5x + 4x) + \sin(5x - 4x) = 2\sin 5x \cos 4x$

(c) By making the substitution $t = \tan\frac{\theta}{2}$, or otherwise, show that **2**
$\operatorname{cosec}\theta + \cot\theta = \cot\frac{\theta}{2}$

(d) (i) If $\sin x - \cos x = A\sin(x - \alpha)$, where α is acute, find A and α. **2**

(ii) Hence or otherwise, solve $\sin x - \cos x = \sqrt{2}$ for $0° \leq x \leq 360°$ **2**

(e)



Two yachts A and B subtend an angle of 60° at the base of a cliff.
From yacht A the angle of elevation of the point P, 100 m vertically above C, is 20°. Yacht B is 600 m from C.

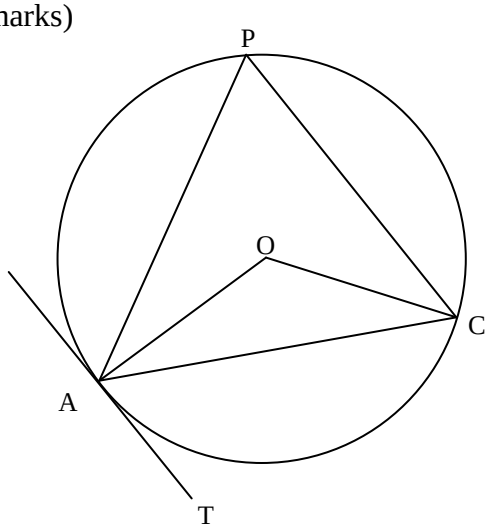
(i) Calculate, the the nearest metre, the length of AC **1**

(ii) Calculate the distance between the two yachts to the nearest metre. **2**

Marks

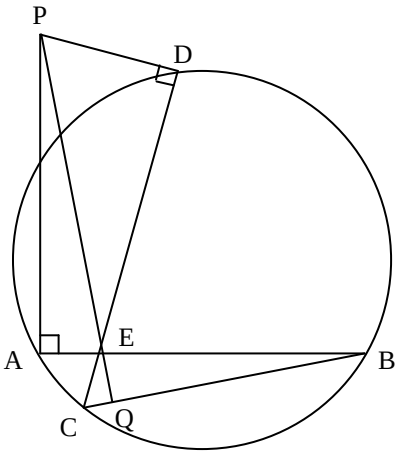
Question 7 (12 marks)

(a)



In the diagram, not drawn to scale, O is the centre of the circle. **3**
AT is a tangent to the circle. $\angle OAB$ is 36°, $\angle CAT$ is $3x°$ and $\angle OCB$ is $x°$.
Copy the diagram onto your answer sheet.
Find the value of x.

(b) Two chords of a circle, AB and CD, intersect at E. The perpendiculars to AB at A and CD at D intersect at P. The line PE meets BC at Q, as shown in the diagram.
Copy the diagram onto your answer sheet.

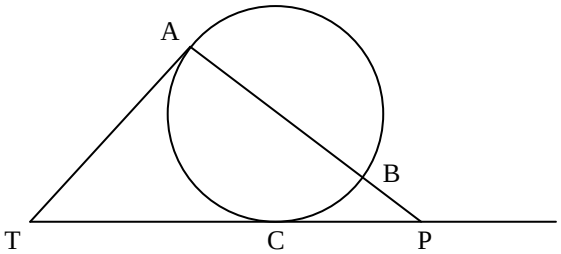


(i) Explain why DPAE is a cyclic quadrilateral. **1**

(ii) Prove that $\angle APE = \angle ABC$ **2**

(iii) Deduce that PQ is perpendicular to BC. **1**

(c)



AB is a diameter of a circle ABC. The tangents at A and C meet at T. The lines TC and AB are produced to meet at P. Copy the diagram and join AC and CB.

(i) Prove that $\angle CAT = 90° - \angle BCP$ **2**

(ii) Hence or otherwise, prove that $\angle ATC = 2\angle BCP$ **3**